



## Improving Parallel Imaging by Jointly Reconstructing Multi-Contrast Data

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## Improving Parallel Imaging by Jointly Reconstructing Multi-Contrast Data

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## ABSTRACT

### Purpose

To develop parallel imaging techniques that simultaneously exploit coil sensitivity encoding, image phase prior information, similarities across multiple images and complementary k-space sampling for highly accelerated data acquisition.

### Methods

We introduce Joint Virtual Coil (JVC-) GRAPPA to jointly reconstruct data acquired with different contrast preparations, and show its application in 2D, 3D and Simultaneous Multi-Slice (SMS) acquisitions. We extend the joint parallel imaging concept to exploit limited support and smooth phase constraints through Joint (J-) LORAKS formulation. J-LORAKS allows joint parallel imaging from limited auto-calibration signal (ACS) region, as well as permitting partial Fourier sampling and calibrationless reconstruction.

### Results

We demonstrate highly accelerated 2D bSSFP with phase-cycling, SMS multi-echo spin echo, 3D multi-echo MPRAGE and multi-echo GRE acquisitions in vivo. Compared to conventional GRAPPA, proposed joint acquisition/reconstruction techniques provide more than 2-fold reduction in reconstruction error.

### Conclusion

JVC-GRAPPA takes advantage of additional spatial encoding from phase information and image similarity, and employs different sampling patterns across acquisitions. J-LORAKS achieves a more parsimonious low rank representation of local k-space by considering multiple images as additional coils. Both approaches provide dramatic improvement in artifact and noise mitigation over conventional single-contrast parallel imaging reconstruction.

## INTRODUCTION

Magnetic Resonance (MR) data acquisition routinely involves image acquisition at multiple echoes or phase-cycles to obtain complementary information. Multi-echo acquisition finds important applications in  $T_2$  and  $T_2^*$  relaxation time mapping (1–4), water/fat imaging (5–8), and reduction of field inhomogeneity related distortion (9). Although enabling numerous applications, achieving whole-brain coverage with high-resolution multi-echo imaging is encoding intensive, leading to excessive scan times.

Another application where multiple images are acquired and combined is balanced steady state free precession (bSSFP). Despite being an SNR efficient sequence with unique  $T_2/T_1$  contrast, bSSFP suffers from image banding artifacts due its sensitivity to  $B_0$  field inhomogeneity. To mitigate these artifacts, multiple images with different RF phase-cycling can be acquired (10,11). This scheme shifts the location of the banding artifacts in each acquisition, so that the phase-cycled images can be combined through e.g. maximum intensity projection (MIP) to eliminate the artifacts. However, collecting multiple phase-cycles increases the scan time and counteracts the inherent efficiency of bSSFP.

Faster acquisitions are possible using receiver encoding, e.g. with sensitivity encoding (12) or generalized auto-calibrating partially parallel acquisitions (GRAPPA) (13). While parallel imaging allows acceleration along one phase encoding direction in 2D acquisitions, undersampling can be flexibly distributed between two axes (phase encoding and partition/slice direction) in 3D (14,15) and SMS imaging (16–20) to achieve higher accelerations.

Parallel imaging can be combined with compressed sensing to exploit sparsity/low-rank properties (21–24), and can be augmented with the Virtual Coil (VC) concept to provide additional spatial encoding using image phase prior information (25–27). On the other hand, LORAKS has been introduced as a novel method that can harness image phase smoothness and limited spatial support, and relies on local low rank properties of k-space to estimate missing data (28). Its extension to parallel imaging also allows utilization of coil sensitivity encoding (29,30). Earlier applications of low rank prior in k- or image-space have also permitted calibrationless parallel imaging (28,29,31–34).

These approaches have been designed to utilize coil sensitivity encoding and prior information to reconstruct a single contrast, without exploiting potential similarities/differences across multiple images. Within the SENSE framework (12), joint reconstruction across echoes/contrasts can be performed by exploiting joint sparse (35–39) (in this context, we use “joint reconstruction” to refer to approaches that couple the reconstruction of multiple images of the same anatomy (37,40)). However, compared to

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3 regularized SENSE per single image (41,42), exploiting similarities at the regularization level was seen to  
4 provide a small improvement (43). Joint reconstruction at the receiver encoding level could serve as a  
5 better alternative to coupling the images at the regularization stage. Such approaches include k-t  
6 GRAPPA (44,45), joint reconstruction of multiple shots in echo-planar diffusion imaging (46,47), k-space  
7 interpolation across all echoes in a gradient and spin echo (GRASE) acquisition (48,49), or across multiple  
8 gradient echoes for temperature mapping (50). Moreover, transmit inhomogeneity at ultra-high field can  
9 be mitigated by acquiring multiple images with different excitation modes, which are then jointly  
10 reconstructed in the TIAMO approach (51).

11  
12 Recent advances in multi-shot diffusion imaging also perform joint reconstruction (46,47). These  
13 techniques aim to reconstruct a single, high-resolution k-space by merging data from multiple  
14 acquisitions while avoiding motion artifacts. Liu et al. (52) made use of self-navigated trajectories to  
15 estimate motion-induced shot-to-shot phase variations. Inverse reconstruction (53) instead solves for  
16 the complex-valued diffusion images in each shot separately. The phase information of each shot is then  
17 used as additional coil sensitivity variation to jointly reconstruct a combined, real-valued diffusion image  
18 with data from all shots. Similarly, MUSE estimates the phase variation of each shot using regularization,  
19 then solves a general model incorporating data from all shots and the calculated phase information (54).  
20 GRAPPA-based realigned kernel techniques (46,47) embed these phase variations into GRAPPA kernels  
21 estimated from additional navigators, which are then used for jointly reconstructing multi-shot data. The  
22 goal of such Joint-GRAPPA DWI techniques is to reduce the sensitivity to mismatches between navigator  
23 and image echoes.

24  
25 MUSSELS is a new approach for phase-calibration-free multi-shot DWI reconstruction (55). This considers  
26 the multi-shot images to have the same contrast, but allow for slowly varying phase across the shots.  
27 These constraints are modeled with an annihilating k-space filter (55–60), which is learned during the  
28 structured low-rank recovery of the missing data.

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30 While multi-shot DW images have the same contrast except for phase discrepancies, we instead focus on  
31 joint reconstruction to treat a broader class of applications, where the acquisitions are made with  
32 multiple contrasts/echoes/cycles. Rather than improved combination of multiple shots, we are targeting  
33 higher acceleration rates. For this, we propose a general framework for joint reconstruction. We  
34 reformulate the joint reconstruction problem as an extension of parallel imaging, and employ existing  
35 components such as GRAPPA, LORAKS, and virtual coils as our building blocks. We also extend the scope,  
36 performance and application space of these techniques. In designing our joint parallel imaging

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3 approaches, our hypothesis was that joint reconstruction would allow us to accelerate multi-contrast  
4 acquisitions further than currently possible with conventional parallel imaging.  
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7 To this end, we introduce the Joint Virtual Coil (JVC) technique wherein multiple echoes/cycles are  
8 reconstructed jointly under the GRAPPA framework. This combines and extends k-t (44,45), realigned  
9 GRAPPA (46,47) and TIAMO (51) approaches with the VC concept (25,26) to permit highly accelerated  
10 2D, 3D and SMS acquisitions. JVC-GRAPPA allows all channels from all image contrasts to contribute to  
11 the reconstruction of a particular channel, and employs VC to convert image phase information into  
12 additional spatial encoding. Data were undersampled with shifts in the k-space sampling pattern across  
13 echoes/cycles to provide complementary k-space coverage and improve reconstruction.  
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17 We further extend the joint parallel imaging concept to exploit limited support and smooth phase  
18 constraints through Joint (J-) LORAKS formulation. J-LORAKS achieves a more parsimonious low rank  
19 representation of local k-space by considering multi-contrast images as additional coils, and allows  
20 reconstruction from limited ACS region. J-LORAKS seamlessly incorporates partial Fourier sampling into  
21 joint parallel imaging and permits improved calibrationless parallel imaging through joint reconstruction.  
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25 Herein, we demonstrate our joint parallel imaging concept in 2D phase-cycled bSSFP, 3D ME-MPRAGE  
26 (9,61) multi-echo gradient-echo (ME-GRE), and SMS multi-echo spin-echo imaging. We have reported  
27 initial versions of this work as abstracts (62,63), where we have shown the application of joint GRAPPA  
28 and SPIRiT (21) in reconstructing phase-cycled bSSFP with 2D and SMS encoding. Herein, we have  
29 extended this initial version with the addition of VC concept, J-LORAKS formalism that admits arbitrary  
30 sampling patterns including partial Fourier and CAIPI (14), calibrationless reconstruction, and application  
31 to multi-echo acquisitions. We also note the elegant profile-encoding by Ilicak et al. that independently  
32 developed joint parallel imaging reconstruction for phase-cycled bSSFP (64,65), which was also extended  
33 to multi-echo acquisition in a recent abstract (66).  
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37 Accompanying Matlab code that reproduces our results is submitted as supplementary material and can  
38 also be downloaded from: <http://bit.ly/2sY1FJT>  
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## 41 42 43 44 45 46 47 48 49 **METHODS**

### 50 51 52 **RECONSTRUCTION ALGORITHMS:**

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54 GRAPPA, JVC-GRAPPA and J-LORAKS were implemented and compared for a number of imaging  
55 cases/applications. All experiments used 16 compressed channels with singular value decomposition  
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(SVD) coil compression for faster reconstruction (67,68). ACS regions used for calibration were included in the final reconstruction for improved SNR and fidelity. Partial Fourier experiments made use of coil-by-coil projection onto convex sets (POCS) processing (69–71) following GRAPPA reconstruction. Experiments were performed on a workstation with 64 Intel Xeon CPU's and 256 GB memory running Matlab 8.0 (Mathworks, Natick, MA). Details of various reconstructions are provided below.

### **GRAPPA and Slice GRAPPA**

Kernel estimation for conventional parallel imaging reconstruction using GRAPPA (13) and Slice GRAPPA (16) was regularized with Tikhonov penalty, and kernel sizes and regularization parameters were selected to minimize root mean squared error (RMSE) relative to the fully sampled data. Slice GRAPPA made use of signal leakage constraint (72) to minimize crosstalk between reconstructed slices.

### **JVC-GRAPPA and JVC Slice GRAPPA**

JVC-GRAPPA creates additional channels by treating data from other echoes/cycles as extra coils. In addition to stacking all contrasts in the coil axis, virtual coil concept is employed to further double the number of channels. Starting with  $N_c$  coils in each of the  $N_e$  echoes, we end up having  $2 \times N_c \times N_e$  total number of channels for joint reconstruction. For example, using typical numbers  $N_c = 16$  and  $N_e = 4$ , the number of coils reach 128, and the amount of kernels that need to be estimated escalates rapidly since this scales with the square of the channel count.

To address this, we follow (26) and perform an iterative procedure where an initial Joint-GRAPPA is performed *without* virtual coils. This way, the entire k-space of the interim reconstruction becomes available for calibration of JVC kernels. We limit the number of JVC iterations to 4, since the gain diminishes after the first couple of iterations (26). In addition to providing ample sample points for kernel estimation, such large calibration region is also better at capturing high-resolution image phase information into the kernels of the virtual coils, thus preventing structured aliasing artifacts (26). During reconstruction, two different Tikhonov regularization parameters were used for the initial Joint ( $\lambda_{\text{init}}$ ) and the latter JVC ( $\lambda_{\text{latter}}$ ) kernel calibrations to further optimize RMSE in the face of increasing calibration region.

To provide complementary frequency information, k-space sampling patterns of the individual echoes/cycles were shifted with respect to each other. Partial Fourier sampling was also explored in 2D, 3D and SMS acquisition settings. This, however, has prevented the use of VC concept because missing

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3 portion of virtual coil k-space due to partial Fourier would be otherwise used for reconstructing the  
4 actual coil k-space based on conjugate symmetry. As such, partial Fourier experiments made use of Joint  
5 (J-) GRAPPA only, without the aid of virtual coils. Similarly, all contrasts were constrained to use the  
6 same partial Fourier sampling direction in the J-GRAPPA reconstructions. To increase the amount of  
7 available k-space region for kernel calibration, J-GRAPPA also used an iterative scheme with 4 iterations.  
8 The initial step used the ACS data to train kernels and generate an interim reconstruction. The following  
9 iteration was then able to utilize the k-space of this interim data and re-train kernels with a larger  
10 calibration region. Again, two different Tikhonov regularization parameters could be used for the initial  
11 and latter iterations.

12  
13 Both regularization parameters, kernel sizes and k-space staggering amounts were optimized to  
14 minimize RMSE in joint reconstruction.

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16 Fig1 provides a depiction of Joint and Joint Virtual Coil GRAPPA reconstructions, ignoring coil and  
17 readout axes for simplicity.

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19 --- Fig1 ---  
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### 23 24 25 26 27 28 29 30 31 32 33 **Autocalibrating J-LORAKS**

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35 J-LORAKS also stacks data from all contrasts in the channel axis, and makes use of image phase  
36 information by creating virtual coils. It enforces local k-space neighborhoods, now extended across all  
37 echoes/cycles in the coil dimension, to have low rank during the reconstruction. There are two  
38 parameters associated with this constraint; the neighborhood size and the target rank of the local k-  
39 space matrices, which were optimized to reduce RMSE. Since J-LORAKS admits arbitrary sampling  
40 patterns, staggering across contrasts, 2D-CAIPI controlled aliasing as well as using different partial  
41 Fourier undersampling (e.g. +k or -k) for each image were explored. While many LORAKS publications  
42 solve non-convex matrix completion problems and are compatible with calibrationless data, substantial  
43 computational accelerations are possible when ACS data is present. Specifically, the autocalibrated  
44 LORAKS framework learns the nullspace properties of the k-space matrices prior to image  
45 reconstruction, and then uses the learned nullspace to formulate image reconstruction as a simple linear  
46 least squares problem that can be solved efficiently (73). Autocalibrating J-LORAKS reconstruction was  
47 performed using preconditioned conjugate gradient (pcg) with 50 iterations for all cases, apart from  
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3 partial Fourier experiments which employed 100 iterations to ensure successful completion of k-space.  
4 Unlike GRAPPA, this also obviated the need for a sequential POCS reconstruction for partial Fourier  
5 sampling. For SMS image reconstruction, autocalibrated J-LORAKS was implemented using the SMS  
6 framework for LORAKS (74).  
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### 10 **Calibrationless J-LORAKS**

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13 In calibrationless J-LORAKS, we assume that no ACS data are available, and thus solve the non-convex  
14 matrix completion problems described in (28,29) instead of the simpler least-squares problem associated  
15 with the autocalibrated case. To reduce reconstruction time, we first reconstruct a central subregion of  
16 the k-space data, and then use that as quasi-ACS data to enable autocalibrated J-LORAKS reconstruction  
17 of progressively larger k-space regions until the entire region has been covered. Since the quasi-ACS  
18 data may have imperfections, we then use the autocalibrated results as an initialization for the original  
19 non-convex optimization problem. The neighborhood size and the matrix rank were tuned to optimize  
20 RMSE. The number of maximum iterations were set to 1000.  
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### 30 **DATA ACQUISITION AND COMPARISON CASES:**

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33 The performances of various reconstruction algorithms were compared for 2D phase-cycled bSSFP, 3D  
34 ME-MPRAGE, SMS multi-echo spin-echo and calibrationless 3D ME-GRE imaging. Imaging parameters  
35 and comparison scenarios are described in detail below.  
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#### 41 **2D Phase-Cycled bSSFP**

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44 **Data Acquisition:** A single abdominal slice of a volunteer was imaged with bSSFP on a 3T Siemens Skyra  
45 system. Four phase-cycles ( $0$ ,  $\pi/2$ ,  $\pi$ ,  $3\pi/2$ ) were collected during a single breath-hold to minimize  
46 motion. Parameters were: field of view (FOV) =  $380 \times 380$  mm<sup>2</sup>, matrix size =  $160 \times 160$ , slice thickness = 5  
47 mm, repetition time (TR) = 3.3 ms, echo time (TE) = 1.54 ms, flip angle =  $37^\circ$ , bandwidth = 822 Hz/pixel,  
48 using 34-channel chest/spine coil reception.  
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53 **Image Reconstruction at 6-fold acceleration:** Fully-sampled data were retrospectively undersampled by  
54  $R=6 \times 1$ -fold with a uniform sampling pattern. The three reconstruction methods used 20 lines of ACS data  
55 for kernel calibration.  
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3 For J-LORAKS, using an ACS size smaller than 20 lines was also explored, and the calibration region was  
4 reduced until J-LORAKS had similar RMSE performance as JVC-GRAPPA from 20 lines of calibration data.  
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6 50 pcg iterations were used in these reconstructions.  
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9 The sampling pattern was shifted by  $\Delta k_y = \{0,1,2,3\}$  samples between the four phase-cycles to provide  
10 complementary k-space coverage in the JVC-GRAPPA and J-LORAKS reconstructions.  
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13 For comparison, VC-GRAPPA *without* joint reconstruction was also performed. As an alternative to k-  
14 space based parallel imaging, Tikhonov-regularized SENSE reconstruction with ESPIRiT coil sensitivity  
15 estimation (75) was also explored. Both the regularization parameter and the threshold for sensitivity  
16 mask size were optimized to reduce RMSE.  
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20 **Image Reconstruction at 7-fold acceleration:** To explore even higher acceleration rates where  
21 conventional parallel imaging would break down, we further pushed the undersampling rate to  $R=7\times 1$ .  
22 We also tested the combination of uniform  $R=6\times 1$ -fold undersampling and partial Fourier acquisition,  
23 while keeping the number of sampled points the same as the  $R=7\times 1$ -fold case. For this, the required  
24 partial Fourier amount to achieve the same number of phase encoding lines, including the ACS data, was  
25  $7/8$ .  
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32 Combination of J-GRAPPA and POCS was used for reconstructing the data at  $R=6\times 1$ -fold undersampling  
33 with  $7/8$  partial Fourier. Following parallel imaging reconstruction, the missing portion of k-space due to  
34 partial Fourier sampling was completed with 100 iterations of POCS.  
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38 J-LORAKS with  $7/8$  partial sampling and  $R=6\times 1$ -fold uniform acceleration employed 100 pcg iterations to  
39 ensure successful completion of partially sampled k-space.  
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### 44 **3D Multi-Echo MPRAGE**

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46 **Data Acquisition:** A volunteer was scanned with a Siemens 3T Skyra system using a fully-sampled ME-  
47 MPRAGE sequence at  $1\text{ mm}^3$  resolution with  $\text{FOV} = 256\times 240\times 192\text{ mm}^3$ . Salient parameters were:  $\text{TR} =$   
48  $2530\text{ ms}$ , inversion time (TI) =  $1100\text{ ms}$ , four echos were sampled at  $\text{TE}'s = \{1.7, 3.6, 5.4, 7.3\}\text{ ms}$ , flip  
49 angle =  $7^\circ$ , and bandwidth =  $651\text{ Hz/pixel}$ . A Siemens 32 channel head coil was used for reception.  
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3 **Image Reconstruction at 12-fold acceleration:** A single slice along the readout direction was selected out  
4 of the 3D dataset, and was retrospectively undersampled along the two phase encoding axes by  $R=4\times 3$ .  
5 Performance of the reconstruction methods was compared using an ACS region size of  $24\times 24$ .  
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9 The sampling pattern was shifted by  $(\Delta k_y, \Delta k_z) = (2,2)$  in the second, by  $(4,4)$  in the third, and by  $(6,6)$   
10 samples in the fourth echo relative to the first TE to provide complementary k-space coverage. In  
11 addition to such complementary sampling, J-LORAKS also employed a different 2D-CAIPI sampling  
12 pattern (14) for each TE to better distribute aliasing. These were designed according to  $(R_y=4, \Delta=0)$  for  
13  $TE_1$ ,  $(R_y=4, \Delta=1)$  for  $TE_2$ ,  $(R_y=4, \Delta=2)$  for  $TE_3$  and  $(R_y=4, \Delta=3)$  for  $TE_4$ .  
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18 Iterative VC-GRAPPA *without* joint reconstruction and Tikhonov-regularized SENSE were also performed  
19 for each echo individually.  
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22 **Image Reconstruction at 16-fold acceleration:** To push the acceleration even further, we compared  
23  $R=4\times 4$  uniform sampling against  $R=4\times 3$ -fold acceleration combined with partial Fourier sampling in both  
24 phase encoding axes. The required partial sampling amount to keep the acquired number of sampled  
25 points the same was  $6/8$ , distributed among  $k_y$  and  $k_z$ .  
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30 Partial Fourier cases were reconstructed using J-GRAPPA with POCS as well as J-LORAKS. J-GRAPPA was  
31 constrained to use the same partial Fourier sampling direction, whereas J-LORAKS used a different partial  
32 Fourier mask for each echo, rotated by  $90^\circ$  in each image, to provide complementary information.  
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### 36 SMS Multi-Echo TSE

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38 **Data Acquisition:** A volunteer was scanned with a Siemens 3T Prisma system using a fully-sampled 2D  
39 multi-echo turbo spin echo (TSE) sequence. The imaging parameters were, FOV =  $240\times 240$ , matrix size =  
40  $256\times 256$ , slice thickness = 4 mm, slice gap = 12.8 mm, number of slices = 10, number of echoes = 6, TR =  
41 4 sec, TE's = {12, 25, 50, 62, 87, 99} ms, echo train length (ETL) = 3, and bandwidth = 260 Hz/pixel. A  
42 Siemens 32 channel product head coil was used for reception.  
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47 **Image Reconstruction at MB-10 acceleration:** The separately encoded 10 slices were retrospectively  
48 collapsed to simulate an MB-10 acquisition. Slice unaliasing performance of conventional and JVC Slice  
49 GRAPPA as well as Joint SMS LORAKS were compared using 24 lines of ACS data.  
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53 **Image Reconstruction at MB-10 acceleration with 6/8 partial Fourier:** The same MB-10 experiment was  
54 performed using an additional  $6/8$  in-plane partial Fourier acceleration. Since VC concept is not  
55 applicable with partial Fourier due to asymmetric sampling, joint parallel imaging was performed with  
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3 Joint Slice GRAPPA, without virtual coils. 100 iterations of POCS were utilized to estimate the missing  
4 data due to partial sampling following conventional and Joint Slice GRAPPA reconstruction.  
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7 Unlike the Joint Slice GRAPPA case, J-SMS-LORAKS still used virtual coils and did not require POCS to  
8 implement phase-constrained partial Fourier reconstruction.  
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### 11 12 13 14 **Calibrationless: 3D Multi-Echo Gradient-Echo**

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17 **Data Acquisition:** A volunteer was scanned using Siemens 3T Skyra system to collect 3D ME-GRE data.  
18 The imaging parameters were, FOV = 240×240×192, matrix size = 160×160×128, TR = 23 ms, TE's = {3, 7,  
19 11, 15, 19} ms, flip angle = 15° and bandwidth = 496 Hz/pixel using a Siemens 32 channel head array.  
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23 **Image Reconstruction at 4-fold acceleration:** A single slice along the readout was taken out from 3D k-  
24 space data. Then, it was retrospectively undersampled with R=4 fold calibrationless Poisson random  
25 sampling pattern in 2D. The performance was compared between conventional single contrast LORAKS  
26 and the proposed multi-contrast J-LORAKS in terms of reconstruction of the individual echoes, as well as  
27 the  $R_2^*$  parameter maps. Echo images were coil combined with the RSoS method, and parameter  
28 mapping was performed by taking the logarithm of the echo images and fitting a line in each voxel. The  
29 negative slope of the fitted line yielded the  $R_2^*$  value in that voxel. To ensure realistic parameter maps, a  
30 non-negativity constraint on the  $R_2^*$  values was applied using the `lsqnonneg` function in Matlab.  
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## 40 **RESULTS**

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42 A quick summary of reconstruction results is provided in Table 2, in which the RMSE performance of the  
43 methods under consideration are compared.  
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### 46 **2D Phase-Cycled bSSFP**

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49 **Image Reconstruction at 6-fold acceleration:** Optimal parameters for conventional GRAPPA were, kernel  
50 size = 7×3 and regularization parameter  $\lambda = 10^{-8}$ . Best RMSE in JVC-GRAPPA was obtained with 5×3  
51 kernels and an initial Joint-GRAPPA reconstruction using regularization parameter  $\lambda_{\text{init}} = 3 \times 10^{-8}$ , which  
52 was increased to  $\lambda_{\text{latter}} = 3 \times 10^{-6}$  in the subsequent JVC iterations. For J-LORAKS, best results were  
53 obtained with a k-space neighborhood radius of 2 voxels and rank constraint = 600.  
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3 Fig2 compares the reconstruction results, where the phase-cycle images were combined with MIP.  
4 Conventional GRAPPA suffered from aliasing artifacts (yellow arrows) and noise amplification and  
5 yielded 13.3% RMSE. Image quality and noise suppression were improved with JVC-GRAPPA, and  
6 reconstruction error was reduced to 7.1%. J-LORAKS provided more than 2-fold RMSE improvement  
7 (6.5%) over conventional GRAPPA using the same calibration region size of 20 lines. Even with a more  
8 stringent calibration region of 16 lines, J-LORAKS had similar performance as JVC-GRAPPA that used 20  
9 ACS lines, (7.1% RMSE, not shown). Fig2 provides further comparison against SENSE (18.6% RMSE) and  
10 VC-GRAPPA (7.5% RMSE) methods, which reconstructed each phase-cycle image separately.

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13 Supporting Fig S1 demonstrates the individual phase-cycles and the sampling patterns, where the  
14 improvement in noise reduction thanks to joint parallel imaging is more apparent. Yellow arrows point to  
15 more subtle aliasing artifacts in JVC-GRAPPA, which were better mitigated in the J-LORAKS  
16 reconstruction.

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25 --- Fig2 ---

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32 **Image Reconstruction at 7-fold acceleration:** Optimal kernel size and regularization parameters at  
33 R=7×1-fold acceleration were 7×3 and  $\lambda = 3 \times 10^{-8}$  for conventional GRAPPA, and 3×3,  $\lambda_{\text{init}} = 3 \times 10^{-8}$  and  
34  $\lambda_{\text{latter}} = 3 \times 10^{-6}$  for JVC-GRAPPA.

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38 At R=6×1-fold undersampling with 7/8 partial Fourier, the optimal parameters for J-GRAPPA and POCS  
39 were, 3×3 kernel size,  $\lambda_{\text{init}} = 3 \times 10^{-8}$  and  $\lambda_{\text{latter}} = 3 \times 10^{-7}$ . J-LORAKS used a local neighborhood of 2 voxels  
40 and rank constraint = 600.

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44 Fig3 shows reconstructed MIPs, error images as well as the k-spaces of the first phase-cycle  
45 reconstructed data. Conventional GRAPPA broke down at such high acceleration with 19.0% RMSE, while  
46 JVC-GRAPPA demonstrated better artifact and noise mitigation and yielded 10.7% error. At the same net  
47 acceleration factor, the combination of 6-fold uniform and 7/8-fold partial Fourier undersampling  
48 returned 9.2% RMSE with J-GRAPPA and POCS. The portion of k-space that was completed with POCS  
49 appeared underestimated (white arrow). J-LORAKS had the best RMSE performance with 8.0%, and  
50 mitigated this underestimation problem.

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--- Fig3 ---

### 3D Multi-Echo MPAGE

**Image Reconstruction at 12-fold acceleration:** The kernel size that yielded the best RMSE was 3×3 for both conventional and JVC-GRAPPA. Optimal Tikhonov parameter for conventional GRAPPA was  $\lambda = 7 \times 10^{-6}$ . For initial Joint GRAPPA reconstruction, regularization parameter was  $\lambda_{\text{init}} = 2 \times 10^{-7}$ , and for the following JVC iterations optimal parameter was  $\lambda_{\text{latter}} = 2 \times 10^{-6}$ .

J-LORAKS obtained optimal performance using a k-space neighborhood of radius = 1 and rank constraint of 300, with 50 pcg iterations.

RSoS combined echoes from the three reconstruction techniques are compared in Fig4. GRAPPA suffered from noise amplification especially in the middle of the FOV (more visible in Supporting Fig S2 where each echo is shown separately). The RMSE of conventional reconstruction was 10.3%, and this was reduced to 6.4% with JVC-GRAPPA. Despite substantially mitigating noise amplification, JVC suffered from a structured aliasing artifact (yellow arrow). This was eliminated in the J-LORAKS reconstruction, with similar noise mitigation and RMSE performance (6.8%). Fig4 also presents comparisons against SENSE and VC-GRAPPA reconstructions that were performed for each echo separately.

Supporting Fig S2 demonstrates the staggered sampling patterns and the individual echoes' reconstructions, where the noise mitigation difference between the conventional and joint techniques can be better appreciated.

--- Fig4 ---

--- Supporting Fig S2 ---

**Image Reconstruction at 16-fold acceleration:** Conventional and JVC-GRAPPA used kernels of size 3×3. Optimal RMSE's were achieved using  $\lambda = 2 \times 10^{-5}$  for conventional GRAPPA, and  $\lambda_{\text{init}} = 2 \times 10^{-7}$ ,  $\lambda_{\text{latter}} = 7 \times 10^{-6}$  for JVC reconstruction.

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3 For the partial Fourier cases, optimal parameters were  $\lambda_{\text{init}} = 2 \times 10^{-7}$ ,  $\lambda_{\text{latter}} = 2 \times 10^{-6}$  for J-GRAPPA, and  
4 neighborhood radius = 1, rank constraint = 300, and 100 pcg iterations for J-LORAKS.  
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7 At this high acceleration factor, conventional GRAPPA demonstrated severe aliasing artifacts and noise  
8 amplification (Fig5), with an RMSE of 14.8%. JVC-GRAPPA partially mitigated these issues with an error of  
9 7.8%, but some aliasing artifacts were still visible. Combination of J-GRAPPA and POCS used R=4x3  
10 uniform and 6/8 partial Fourier undersampling to achieve the same net acceleration factor. Despite an  
11 overall improvement in artifact reduction, partially sampled k-space suffered from underestimation  
12 (white arrow) and some structured aliasing artifacts were present (yellow arrow). J-LORAKS was able to  
13 further address these issues to provide a cleaner reconstruction with an RMSE of 7.9%.  
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23 --- Fig5 ---  
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## 28 SMS Multi-Echo Spin-Echo

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30 **Image Reconstruction at MB-10 acceleration:** FOV shift between slices was optimized to yield the best  
31 RMSE and was found to be FOV/4.  
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34 Optimal kernel size and Tikhonov regularization parameter for conventional Slice GRAPPA were 9x9 and  
35  $10^{-6}$ . These were selected as 7x7 and  $10^{-7}$  for JVC Slice GRAPPA.  
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39 The parameters chosen for J-SMS-LORAKS were neighborhood radius = 3 (which lead to circle diameter  
40 7), rank  $r = 1000$ , with regularization parameter  $\lambda = 10^{-5}$ .  
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43 RSoS combination of reconstructed echoes are shown in Fig6. Conventional Slice GRAPPA yielded 5.1%  
44 error and exhibited structured aliasing artifacts (yellow arrows). JVC Slice GRAPPA partially mitigated  
45 these artifacts as well as reducing the noise amplification (better appreciated in Supporting Fig S3 where  
46 individual echo images are shown). While the RMSE was reduced to 3.6%, some structured artifacts were  
47 present (yellow arrows). J-SMS-LORAKS was more successful at artifact mitigation, as well as at RMSE  
48 performance (3.3%).  
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53 --- Fig6 ---  
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3 --- Supporting Fig S3 ---  
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6 Whereas Fig6 displays only 5 out of 10 reconstructed slices, the entire 10-slice reconstruction can be  
7 viewed in Supporting Fig S4.  
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15 **Image Reconstruction at MB-10 acceleration with 6/8 partial Fourier:** All reconstruction parameters for  
16 the three methods were the same as the previous MB-10 experiment.  
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19 Reconstruction results, error images and k-space data of the first echo are compared in Fig7. Slice  
20 GRAPPA followed by POCS processing returned 6.0% error with aliasing artifacts as pointed by yellow  
21 arrows. Due to asymmetric k-space sampling, VC concept could not be exploited in Joint Slice GRAPPA.  
22 As such, the reconstruction was performed without the aid of virtual coils, and sequential POCS  
23 processing was applied to estimate the partially sampled portion. Despite the reduction in RMSE to 4.9%,  
24 some residual aliasing artifacts were still present (yellow arrows). Apart from the readout line at the  
25 edge of the partial Fourier sampling mask, POCS completed portion did not appear to be  
26 underestimated. This is because the background phase is minimal for spin-echo data, unlike the bSSFP  
27 and MPRAGE cases.  
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35 J-SMS-LORAKS attained the best RMSE performance (3.7%), with some structured aliasing artifacts at  
36 such high acceleration (yellow arrows). Estimated k-space appeared smooth, and devoid of discontinuity  
37 or underestimation problems.  
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41 --- Fig7---  
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44 For display purposes, Fig7 shows only 5 out of 10 slices reconstructed in this MB10 experiment. The  
45 entire array of 10 slices can be viewed in Supporting Fig S5.  
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49 --- Supporting Fig S5 ---  
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54 **Calibrationless: 3D Multi-Echo Gradient-Echo**  
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3 The parameters were tuned to minimize RMSE, where neighborhood radius = 3 for both cases, and the  
4 matrix rank  $r = 50$  for single contrast and  $r = 300$  for multi-contrast LORAKS. While both reconstructions  
5 were devoid of visible artifacts, observing the error maps in Fig8 revealed a signal bias especially in the  
6 early echoes of the conventional LORAKS results. These were reflected in the estimated  $R_2^*$  parameter  
7 maps, where J-LORAKS mitigated the underestimation problem that single-LORAKS suffered from (yellow  
8 arrow). The reconstruction errors were 4.2% and 3.1% for the two algorithms.  
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## 23 DISCUSSION

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25 We presented joint parallel imaging acquisition/reconstruction approaches that exploit similarities  
26 between multi-contrast images, as well as complementary sampling and image phase priors to provide  
27 dramatic improvements over conventional techniques for highly accelerated acquisitions. JVC-GRAPPA  
28 made this possible by converting intensity and phase differences across images into extra spatial  
29 encoding and utilizing the VC concept. J-LORAKS, sought to achieve local k-space matrices with lower  
30 rank because of the added redundancy from the multiple images stacked along the coil axis. Proposed  
31 joint reconstruction techniques thus enabled acceleration rates beyond the capability of conventional  
32 parallel imaging, while mitigating aliasing artifacts and reducing noise amplification.  
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39 JVC-GRAPPA is a straightforward extension of GRAPPA, where multi-contrast images are stacked in the  
40 coil axis with staggered k-space sampling and iterative VC reconstruction. J-LORAKS addresses two main  
41 limitations of JVC-GRAPPA: (i) it allows arbitrary sampling patterns, and (ii) can work with parsimonious  
42 ACS size or even without calibration. We took advantage of (i) by using different sampling patterns in  
43 each contrast, as well as exploiting VC concept despite partial Fourier sampling. Mitigating drawback (ii)  
44 is especially important at high acceleration rates, where the span of GRAPPA kernels can be very large.  
45 At  $R=7$ -fold acceleration with a small kernel size of 3 samples, GRAPPA kernels would already span a 15-  
46 sample distance in k-space. With small ACS sizes, it becomes difficult to extract sufficient amount of  
47 training data because we can only slide such a large 15-sample window in k-space by a few samples. This  
48 also constrains the size of the GRAPPA kernel, e.g. we would need 30 ACS lines to be able to fit a 5-  
49 sample kernel.  
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3 In terms of reconstruction time, both algorithms perform similarly with a small advantage for J-LORAKS.  
4 Points where JVC-GRAPPA might be advantageous are its simplicity in implementation and less memory  
5 usage than J-LORAKS. Overall, J-LORAKS is superior to JVC-GRAPPA in most aspects, except for the  
6 relative ease of exporting it to online reconstruction platforms.  
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10 Joint reconstruction of calibrationless dataset showed advantages in RMSE (Fig8), which could be further  
11 improved using different sampling patterns across the echoes. In most conventional exams, calibration  
12 signal can be acquired by fully-sampling the k-space center, or by using a separate acquisition e.g. a low-  
13 resolution GRE. Since such separate calibration information is cheap, Cartesian acquisitions for high-  
14 resolution structural imaging may not benefit from calibrationless reconstruction.  
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20 There are, however, applications where a rapid separate acquisition may not provide suitable calibration  
21 information, or an integrated ACS region with Nyquist-sampling may be disadvantageous. For instance,  
22 inconsistencies between the separate ACS data and the accelerated functional imaging acquisition can  
23 reduce the temporal SNR (76), and it is not practical to have an integrated calibration region in echo  
24 planar trajectories. Dynamic imaging could be another domain where calibrationless reconstruction  
25 could be impactful (33), since coil sensitivity information is subject to change due to motion and it is  
26 costly to sample ACS data over time. Despite having potential applications, increased computational  
27 burden and the reduction in the achievable acceleration are some of the trade-offs in calibrationless  
28 imaging.  
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36 Because SENSE-based reconstruction does not introduce coupling between different contrasts, we have  
37 chosen to employ GRAPPA and LORAKS in joint reconstruction. However, we have combined SENSE and  
38 LORAKS in our earlier work (30), and recently improved on this by combining LORAKS with k-space  
39 parallel imaging constraints in (58,59). In particular, (58) provides a comparison of SENSE-based LORAKS  
40 versus autocalibrating LORAKS, where the latter has a clear advantage.  
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45 We think that VC-GRAPPA can be modified to allow partial Fourier reconstruction. Similar to the way that  
46 the GRAPPA kernel needs to change for each distinct local sampling pattern, we would need to apply a  
47 different GRAPPA kernel for the asymmetrically sampled region compared to the kernel used for the  
48 symmetric region. This kernel for the asymmetric region would employ only the actual k-space during  
49 the training and reconstruction stages. For simplicity, we used J-GRAPPA without VC in partial Fourier  
50 experiments. The impact of not using VC can be appreciated by comparing Figs6&7. With JVC Slice  
51 GRAPPA, reconstruction error reduced by 42% compared to Slice GRAPPA (Fig6, no partial-Fourier case).  
52 With Joint Slice GRAPPA (Fig7, 6/8 partial-Fourier, no VC) the improvement was 22%. With J-LORAKS, the  
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3 phase prior constraint is better incorporated into the reconstruction for partial Fourier acquisition to  
4 provide a 62% reduction compared to standard GRAPPA reconstruction. Please also see Supporting  
5 Information for further discussion on partial Fourier sampling in specific acquisitions. We have also  
6 explored using VC concept *without* joint reconstruction in Supplementary Figs S2&4. This has provided  
7 improvement over conventional GRAPPA/SENSE, but failed to reach the quality of the proposed joint  
8 reconstruction algorithms.  
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14 Shifted sampling patterns are helpful in providing increased collective frequency coverage. In the bSSFP  
15 experiments at R=6-fold acceleration (Fig2), JVC-GRAPPA would have yielded 8.4% RMSE if all phase-  
16 cycles were using the same undersampling pattern, compared to the optimal 7.1% with the staggered  
17 acquisition. For J-LORAKS, the reconstruction error would have increased to 7.0%, as opposed to the  
18 optimal 6.5% RMSE. These indicate that complementary undersampling is aiding the joint reconstruction,  
19 but both algorithms are robust to changes in the sampling strategy, with small degradation in the RMSE  
20 performance (less than 20%). Table 1 report the same analysis for MPRAGE reconstruction, where both  
21 techniques are seen to be robust to changes in the staggering amount (less than 15% degradation  
22 without shifts).  
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30 Because it stacks data from all image contrasts into the channel axis and also creates virtual coils, JVC-  
31 GRAPPA requires larger amount of calibration data as the number of kernels scale with the square of the  
32 channel count. This problem is exacerbated by shifted sampling, as the staggered JVC kernels span a  
33 larger k-space extent which is more difficult to fit in the ACS region. This is addressed in part by the  
34 iterative approach, which uses the reconstructed k-space to re-train the kernels. Increasing the size of  
35 calibration region however lengthens the reconstruction time, since the calibration matrix that needs to  
36 be inverted grows in size. J-LORAKS addresses this second limitation of large ACS requirement as well,  
37 and is capable of outperforming JVC-GRAPPA even when it uses a smaller calibration region size (Fig1).  
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44 The reconstruction errors in the 2D and 3D acquisition experiments include some contribution from the  
45 intrinsic  $\sqrt{R}$  SNR penalty. This stems from the data undersampling that reduces the total noise averaging  
46 window of the images. Such  $\sqrt{R}$  penalty does not impact SMS acquisition in practice, since SMS k-space  
47 data are not undersampled and each slice experiences the same noise averaging benefit as the MB-1  
48 case. However, because we have simulated SMS acceleration by collapsing separately acquired slices,  
49 our results are actually impacted by  $\sqrt{MB}$  =  $\sqrt{10}$  noise penalty in Figs6&7. Since the actual SMS  
50 experiment would not have been affected by this additional noise, the RMSE levels would have been  
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3 lower. Nonetheless, it would not be possible to eliminate the additional factor of  $\sqrt{4/3}$  SNR penalty due  
4 to partial Fourier undersampling in Fig7.  
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#### 8 9 **Limitations:**

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11 We have employed different regularization parameters for kernel training in the initial Joint-GRAPPA  
12 reconstruction and the following JVC iterations for optimal RMSE. It is possible to use the same Tikhonov  
13 parameters for the two steps without a large drop in the performance. For instance, using  $\lambda_{\text{init}} = \lambda_{\text{latter}} =$   
14  $3 \times 10^{-8}$  in Fig2 led to an RMSE of 7.7%, which is slightly higher than the optimal JVC performance (7.1%).  
15 Automatic parameter selection algorithms could further help address this limitation (77,78).  
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21 Another drawback in the joint parallel imaging reconstruction is the increased reconstruction time. For  
22 the results presented in Fig2, computation time for the four phase-cycles was 6 sec for GRAPPA, 6.8 min  
23 for JVC-GRAPPA, and 5.4 min for J-LORAKS. We think that there are several ways to reduce this 50-fold  
24 gap in performance. We have used SVD coil compression in the current experiments. More advanced  
25 compression techniques such as Geometric Coil Compression (79) would permit higher compression  
26 rates. Secondly, JVC-GRAPPA uses the entire k-space to re-train kernels in the latter iterations. This  
27 calibration size could be restricted to a smaller portion of k-space to reduce the calibration time, at the  
28 cost of increasing the condition number of the matrix inversion. Rather than applying these large  
29 number of JVC kernels via convolution in k-space, an image space version could be implemented with a  
30 simple elementwise multiplication. Finally, the number of pcg iterations in J-LORAKS could be reduced to  
31 reach a better compromise between speed and accuracy.  
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41 A further limitation of the joint reconstruction is potential motion between scans. While multi-echo  
42 acquisitions do not suffer from this drawback, phase-cycled bSSFP could be impacted by potential  
43 mismatches across cycles. Since JVC-GRAPPA employs low-resolution kernels for data interpolation, we  
44 expect this technique to be resilient against small amounts of motion. In the presence of larger  
45 mismatches, an initial GRAPPA could be applied on each phase-cycle independently, followed by  
46 retrospective motion correction and JVC-GRAPPA or J-LORAKS processing. The higher acceleration  
47 factors that can be achieved with joint reconstruction could also help mitigate some of the involuntary  
48 motion.  
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**Extensions:**

A potential extension to the proposed joint parallel imaging techniques could be the addition of joint sparse regularization (36,43). This extension would easily fit within SPIRiT (21) or LORAKS (28) frameworks. For JVC-GRAPPA, sparsity enforcing priors could be used either during kernel calibration (80) or reconstruction (23).

Another application where joint parallel imaging could be powerful is single-shot diffusion imaging, where multiple diffusion volumes at neighboring q-space positions could be jointly reconstructed.

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## TABLES

RMSE(JVC-GRAPPA): MPRAGE @ R=12				RMSE(J-LORAKS): MPRAGE @ R=12					
		$\Delta k_z$					$\Delta k_z$		
		0	1	2			0	1	2
$\Delta k_y$	0	7.3%	6.7%	6.7%	$\Delta k_y$	0	7.1%	<b>6.8%</b>	<b>6.8%</b>
	1	6.6%	6.6%	6.6%		1	7.0%	<b>6.8%</b>	<b>6.8%</b>
	2	<b>6.4%</b>	<b>6.4%</b>	<b>6.4%</b>		2	7.0%	<b>6.8%</b>	<b>6.8%</b>

**Table1.** Performance analysis of 12-fold accelerated JVC-GRAPPA and J-LORAKS reconstructions for multi-echo MPRAGE acquisition with different k-space staggering amounts across echoes.

	RMSE%			
	(Slice) GRAPPA	JVC (Slice) GRAPPA	Joint (Slice) GRAPPA & POCS	Joint (SMS) LORAKS
Phase-cycled bSSFP @ R=6	13.3%	7.1%	–	<b>6.5%</b>
Phase-cycled bSSFP @ R=7	19.0%	10.7%	9.2%	<b>8.0%</b>
ME-MPRAGE @ R=12	10.3%	<b>6.4%</b>	–	6.8%
ME-MPRAGE @ R=16	14.8%	<b>7.8%</b>	8.5%	7.9%
ME-TSE @ MB=10	5.1%	3.6%	–	<b>3.3%</b>
ME TSE @ MB=10 & PF=6/8	6.0%	–	4.9%	<b>3.7%</b>

**Table2.** Comparison of GRAPPA and the proposed joint reconstruction algorithms, where the results in Figs 2-7 are summarized.

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60**FIGURE CAPTIONS**

**Fig 1.** Joint GRAPPA fits kernels across contrasts and employs staggered k-space sampling to improve parallel imaging capability. Joint Virtual Coil GRAPPA further employs extra phase information provided by virtual coils to synthesize the target k-space signal.

**Fig2.** Maximum intensity projection combination of phase-cycled bSSFP reconstructions with four cycles at R=6x1-fold acceleration. Conventional SENSE and GRAPPA suffered from noise amplification and aliasing artifacts, leading to 18.6% and 13.3% RMSE, respectively. VC-GRAPPA made use of image phase information to improve the reconstruction with 7.5% RMSE. JVC-GRAPPA jointly reconstructed the phase-cycles with staggered k-space sampling and mitigated noise and aliasing with 7.1% error. J-LORAKS further improved the image quality and RMSE performance (6.5%). Even when using a more limited calibration region of 16 samples, J-LORAKS was able to yield similar performance as JVC-GRAPPA using 20 lines of ACS (7.1% RMSE, not shown).

**Fig3.** Phase-cycled bSSFP with 7-fold total acceleration. Conventional GRAPPA broke down at such high acceleration factor, and had 19.0% error with severe aliasing artifacts and noise amplification. JVC-GRAPPA was able to mitigate some of these artifacts, but still yielded a large error of 10.7%. Combination of 6-fold uniform and 7/8 partial Fourier sampling provided the same 7-fold net acceleration. In this setting, virtual coil concept was not applicable in Joint GRAPPA. Its combination with

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3 POCS reconstruction led to 9.2% RMSE and some signal underestimation in k-space (white arrow). J-  
4 LORAKS was able to outperform all methods with 8.0% error, and was more successful in completing the  
5 partially sampled k-space without the need for an additional POCS step.  
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9 **Fig4.** Root-sum-of-squares combination of multi-echo MPRAGE reconstructions with four echoes and  
10 R=4x3-fold acceleration. Regularized SENSE reconstruction suffered from structured artifacts with 9.9%  
11 error. Conventional GRAPPA had noise amplification especially in the middle of the field of view with  
12 10.3% RMSE. VC-GRAPPA provided minor improvement to yield 9.5% error. JVC-GRAPPA mitigated this  
13 and reduced the error to 6.4%, but at the expense of some structured aliasing artifact (yellow arrow). J-  
14 LORAKS provided a cleaner image with reduced noise amplification and 6.8% error.  
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20 **Fig5.** Multi-echo MPRAGE reconstruction at 16-fold total acceleration. Conventional GRAPPA broke down  
21 at this high acceleration factor, yielding 14.8% error. JVC-GRAPPA managed to mitigate most of the  
22 structured artifacts and noise amplification, with 7.8% RMSE and some residual aliasing artifacts (yellow  
23 arrow). At the same net acceleration factor, the combination of R=4x3 uniform undersampling and 6/8  
24 partial Fourier sampling was explored. Combination of J-GRAPPA and POCS had 8.5% error,  
25 underestimation in partially sampled k-space (white arrow), and structured artifact (yellow arrow). J-  
26 LORAKS was able to provide an improved reconstruction with 7.9% RMSE and more successfully  
27 completed partial k-space data.  
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34 **Fig6.** Multi-echo Turbo Spin Echo reconstruction with six echoes and Simultaneous MultiSlice  
35 acceleration (displaying 5 out of 10 slices). Root-sum-of-squares combination of the echoes using  
36 MultiBand=10 acceleration are depicted. Conventional Slice GRAPPA with signal leakage constrained  
37 yielded 5.1% error and structured aliasing artifacts (yellow arrows). JVC Slice GRAPPA obtained a reduced  
38 RMSE of 3.6% with better noise suppression and artifact mitigation. However, some aliasing artifacts  
39 were still visible. Joint SMS LORAKS provided the lowest error 3.3% with better artifact suppression.  
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45 **Fig7.** Multi-echo Turbo Spin Echo reconstruction with MultiBand=10 acceleration and 6/8 partial Fourier  
46 sampling (displaying 5 out of 10 slices). Combination of Slice GRAPPA and POCS had 6.0% RMSE with  
47 visible aliasing artifacts and some k-space discontinuity at the partial Fourier transition line. Joint Slice  
48 GRAPPA was not able to utilize virtual coil concept, and required POCS post-processing to estimate  
49 partially sampled data. This combination led to 4.9% error with some reconstruction artifacts and minor  
50 k-space discontinuity. Joint SMS LORAKS did employ virtual coils, and incorporated partial Fourier  
51 reconstruction without the need for POCS processing. This allowed 3.7% RMSE performance, while not  
52 being able to fully mitigate aliasing artifacts at such high acceleration factor (yellow arrows).  
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3 **Fig8.** Calibrationless multi-echo gradient-echo reconstruction at R=4-fold pseudo-random acceleration.  
4 Single-contrast calibrationless LORAKS yielded 4.2% error with underestimation in the  $R_2^*$  parameter  
5 map (yellow arrow). This is likely caused by the signal drop in the early echoes as can be better seen in  
6 the error maps. Joint calibrationless LORAKS had an improved RMSE performance of 3.1%, and mitigated  
7 the signal dropout problem in both the individual echoes and the estimated parameter map.  
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### 12 SUPPORTING FIGURE CAPTIONS

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15 **Supporting Fig S1.** Individual bSSFP phase-cycle images at R=6-fold acceleration. K-space sampling  
16 patterns are indicated below each phase-cycle. Conventional GRAPPA incurred noise amplification and  
17 structured aliasing artifacts with 13.3% RMSE. These were largely mitigated in JVC-GRAPPA with 7.1%  
18 error, while some subtle artifacts remained (yellow arrows). J-LORAKS further improved the image  
19 quality to yield 6.5% RMSE.  
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24 **Supporting Fig S2.** Individual echoes from the R=12-fold accelerated ME-MPRAGE reconstruction.  
25 GRAPPA with uniform sampling led to substantial noise amplification and 10.3% error. JVC-GRAPPA used  
26 staggered sampling across echoes, reduced the noise amplification and improved the RMSE to 6.4%, but  
27 suffered from some structured aliasing artifacts (yellow arrows). J-LORAKS explored the use of different  
28 CAIPI sampling patterns to mitigate both noise amplification and aliasing artifacts, with similar RMSE  
29 performance (6.8%).  
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35 **Supporting Fig S3.** Individual echoes from SMS multi-echo TSE reconstruction at MB-10 acceleration.  
36 Slice GRAPPA achieved 5.1% RMSE and suffered from noise amplification and aliasing artifacts (yellow  
37 arrows), which became less apparent at the later echoes due to relaxation. JVC Slice GRAPPA was able to  
38 mitigate the noise amplification, but some of the visible aliasing artifacts remained (3.6% RMSE). Joint  
39 SMS Loraks further improved the image quality and attained the lowest reconstruction error of 3.3%.  
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44 **Supporting Fig S4.** Multi-echo SMS TSE results at MB-10 acceleration, displaying all 10 slices from the  
45 three reconstruction techniques.  
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47 **Supporting Fig S5.** Multi-echo SMS TSE results at MB-10 acceleration and 6/8 Partial Fourier sampling. In  
48 addition to all 10 of the reconstructed slices, corresponding error images and k-space representation are  
49 displayed for the three algorithms.  
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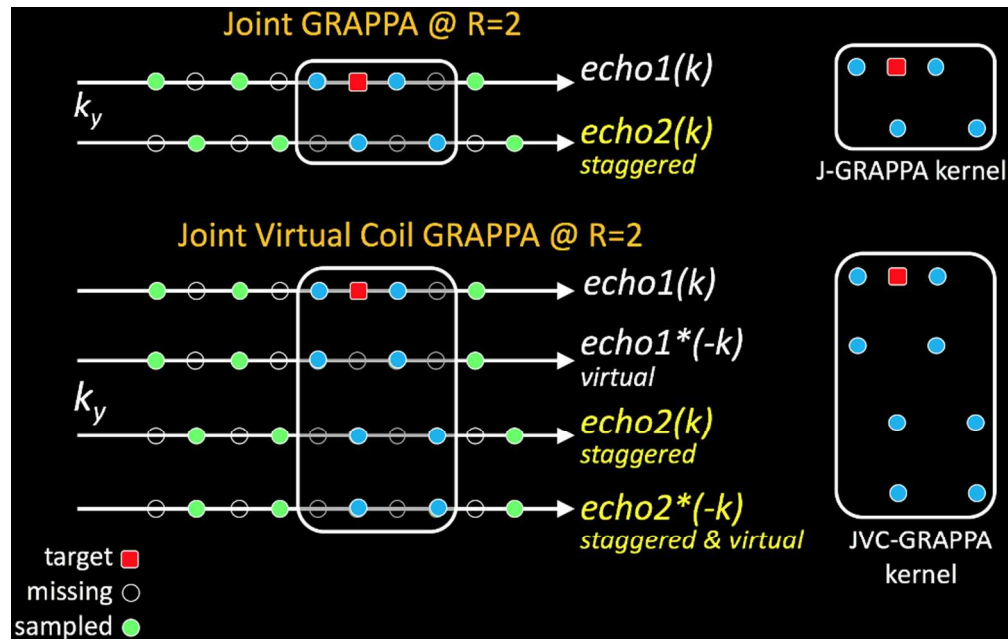
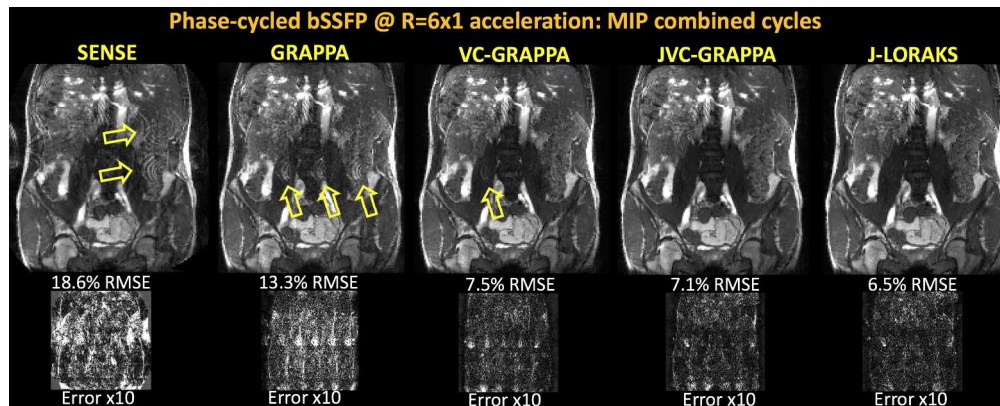


Fig 1. Joint GRAPPA fits kernels across contrasts and employs staggered k-space sampling to improve parallel imaging capability. Joint Virtual Coil GRAPPA further employs extra phase information provided by virtual coils to synthesize the target k-space signal.

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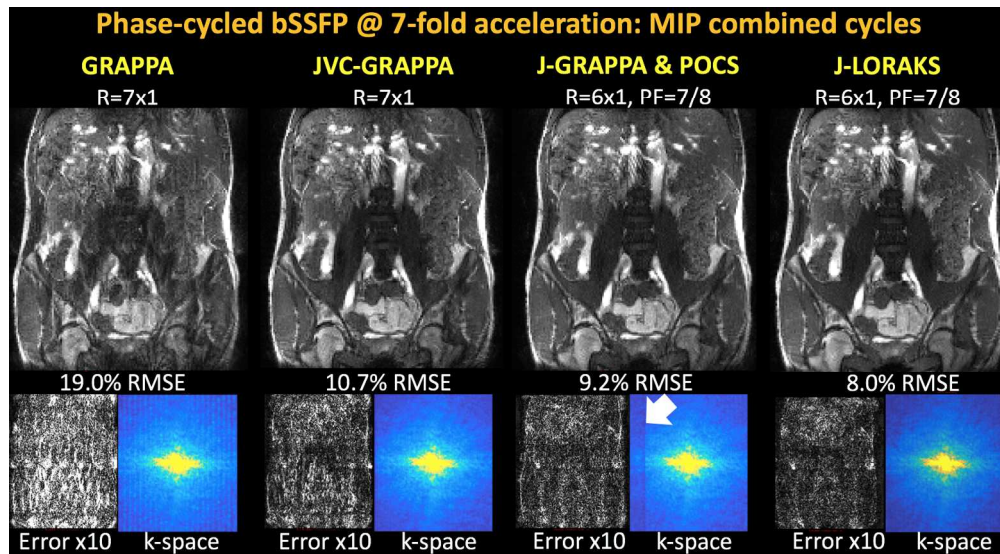
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Fig2. Maximum intensity projection combination of phase-cycled bSSFP reconstructions with four cycles at R=6x1-fold acceleration. Conventional SENSE and GRAPPA suffered from noise amplification and aliasing artifacts, leading to 18.6% and 13.3% RMSE, respectively. VC-GRAPPA made use of image phase information to improve the reconstruction with 7.5% RMSE. JVC-GRAPPA jointly reconstructed the phase-cycles with staggered k-space sampling and mitigated noise and aliasing with 7.1% error. J-LORAKS further improved the image quality and RMSE performance (6.5%). Even when using a more limited calibration region of 16 samples, J-LORAKS was able to yield similar performance as JVC-GRAPPA using 20 lines of ACS (7.1% RMSE, not shown).

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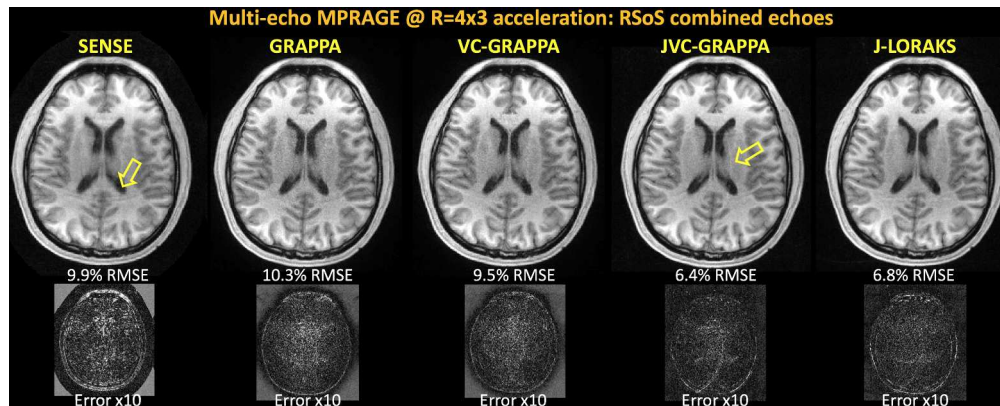


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Fig3. Phase-cycled bSSFP with 7-fold total acceleration. Conventional GRAPPA broke down at such high acceleration factor, and had 19.0% error with severe aliasing artifacts and noise amplification. JVC-GRAPPA was able to mitigate some of these artifacts, but still yielded a large error of 10.7%. Combination of 6-fold uniform and 7/8 partial Fourier sampling provided the same 7-fold net acceleration. In this setting, virtual coil concept was not applicable in Joint GRAPPA. Its combination with POCS reconstruction led to 9.2% RMSE and some signal underestimation in k-space (white arrow). J-LORAKS was able to outperform all methods with 8.0% error, and was more successful in completing the partially sampled k-space without the need for an additional POCS step.

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Fig4. Root-sum-of-squares combination of multi-echo MPRAGE reconstructions with four echoes and R=4x3-fold acceleration. Regularized SENSE reconstruction suffered from structured artifacts with 9.9% error. Conventional GRAPPA had noise amplification especially in the middle of the field of view with 10.3% RMSE. VC-GRAPPA provided minor improvement to yield 9.5% error. JVC-GRAPPA mitigated this and reduced the error to 6.4%, but at the expense of some structured aliasing artifact (yellow arrow). J-LORAKS provided a cleaner image with reduced noise amplification and 6.8% error.

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938x376mm (72 x 72 DPI)

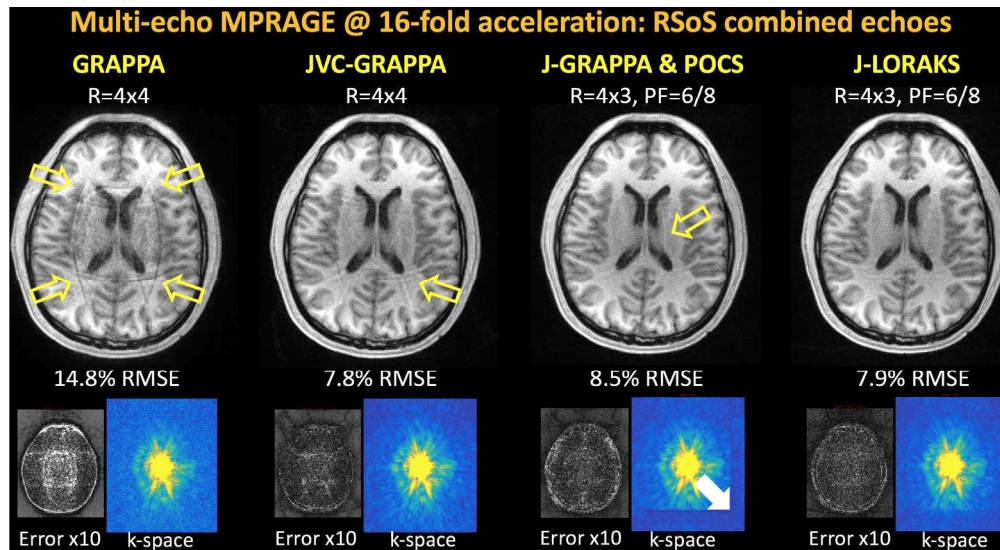
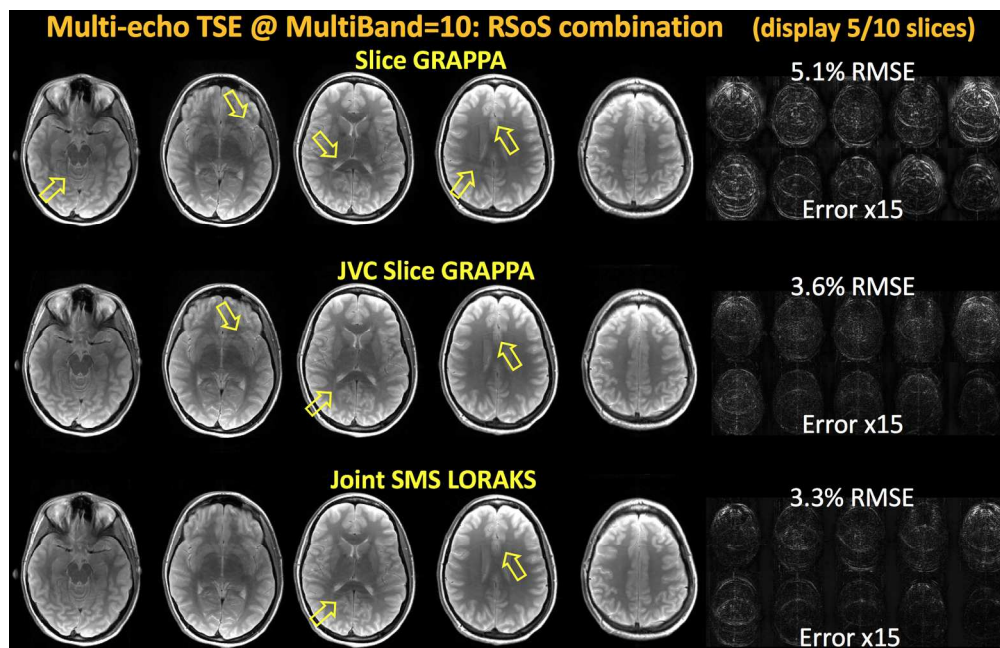


Fig5. Multi-echo MPRAGE reconstruction at 16-fold total acceleration. Conventional GRAPPA broke down at this high acceleration factor, yielding 14.8% error. JVC-GRAPPA managed to mitigate most of the structured artifacts and noise amplification, with 7.8% RMSE and some residual aliasing artifacts (yellow arrow). At the same net acceleration factor, the combination of R=4x3 uniform undersampling and 6/8 partial Fourier sampling was explored. Combination of J-GRAPPA and POCS had 8.5% error, underestimation in partially sampled k-space (white arrow), and structured artifact (yellow arrow). J-LORAKS was able to provide an improved reconstruction with 7.9% RMSE and more successfully completed partial k-space data.

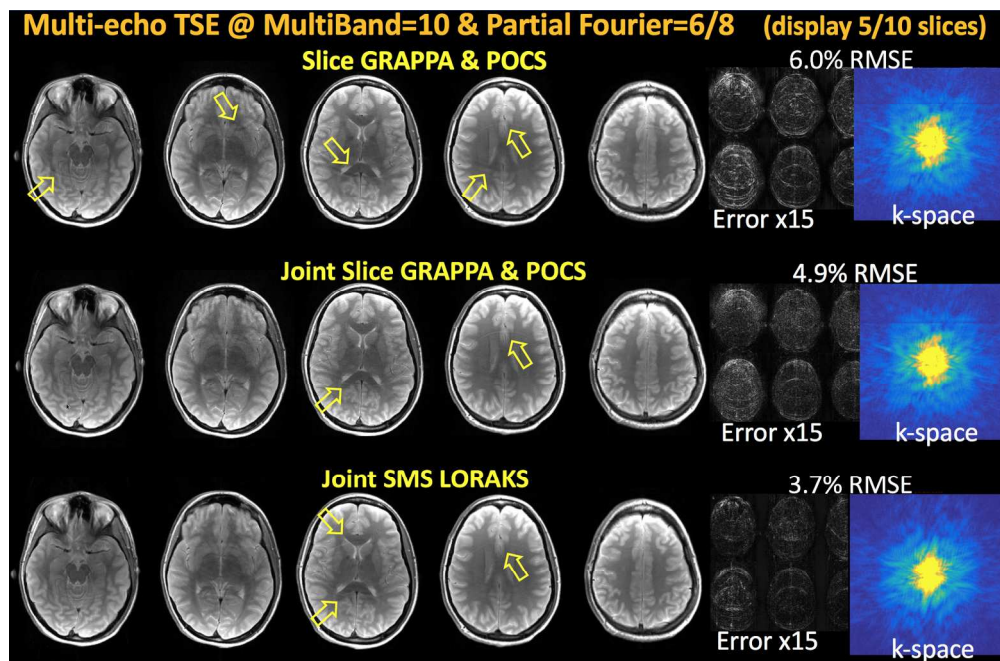
175x95mm (300 x 300 DPI)



28 Fig6. Multi-echo Turbo Spin Echo reconstruction with six echoes and Simultaneous MultiSlice acceleration  
29 (displaying 5 out of 10 slices). Root-sum-of-squares combination of the echoes using MultiBand=10  
30 acceleration are depicted. Conventional Slice GRAPPA with signal leakage constrained yielded 5.1% error  
31 and structured aliasing artifacts (yellow arrows). JVC Slice GRAPPA obtained a reduced RMSE of 3.6% with  
32 better noise suppression and artifact mitigation. However, some aliasing artifacts were still visible. Joint SMS  
33 LORAKS provided the lowest error 3.3% with better artifact suppression.

34 175x112mm (300 x 300 DPI)

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Fig7. Multi-echo Turbo Spin Echo reconstruction with MultiBand=10 acceleration and 6/8 partial Fourier sampling (displaying 5 out of 10 slices). Combination of Slice GRAPPA and POCS had 6.0% RMSE with visible aliasing artifacts and some k-space discontinuity at the partial Fourier transition line. Joint Slice GRAPPA was not able to utilize virtual coil concept, and required POCS post-processing to estimate partially sampled data. This combination led to 4.9% error with some reconstruction artifacts and minor k-space discontinuity. Joint SMS LORAKS did employ virtual coils, and incorporated partial Fourier reconstruction without the need for POCS processing. This allowed 3.7% RMSE performance, while not being able to fully mitigate aliasing artifacts at such high acceleration factor (yellow arrows).

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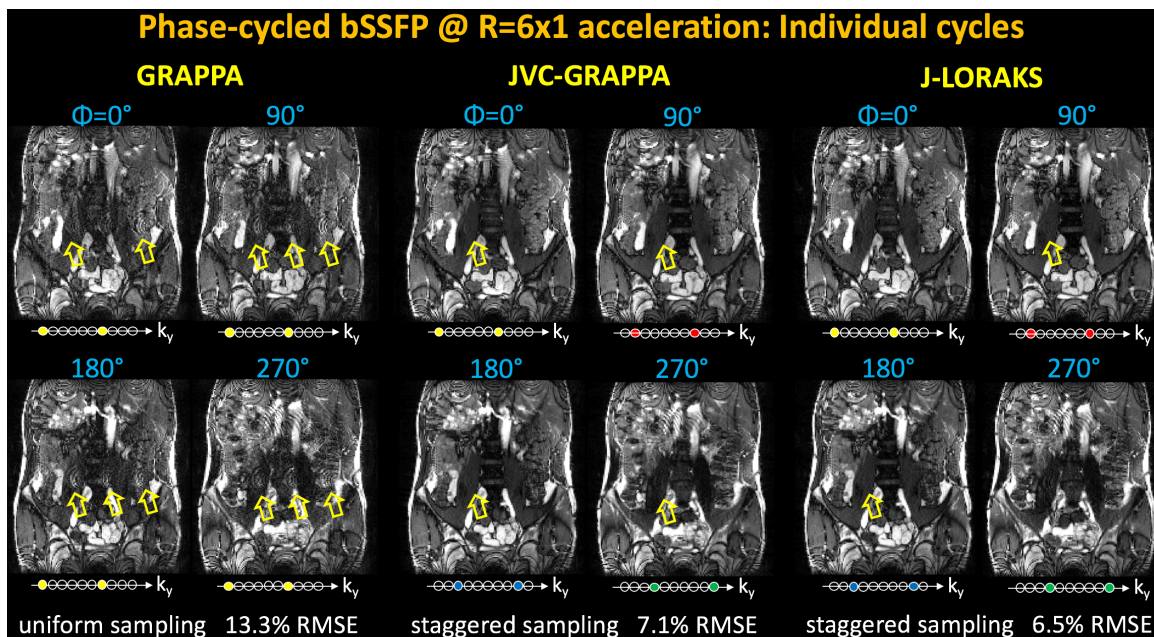


## Supporting discussion on partial Fourier sampling in specific sequences

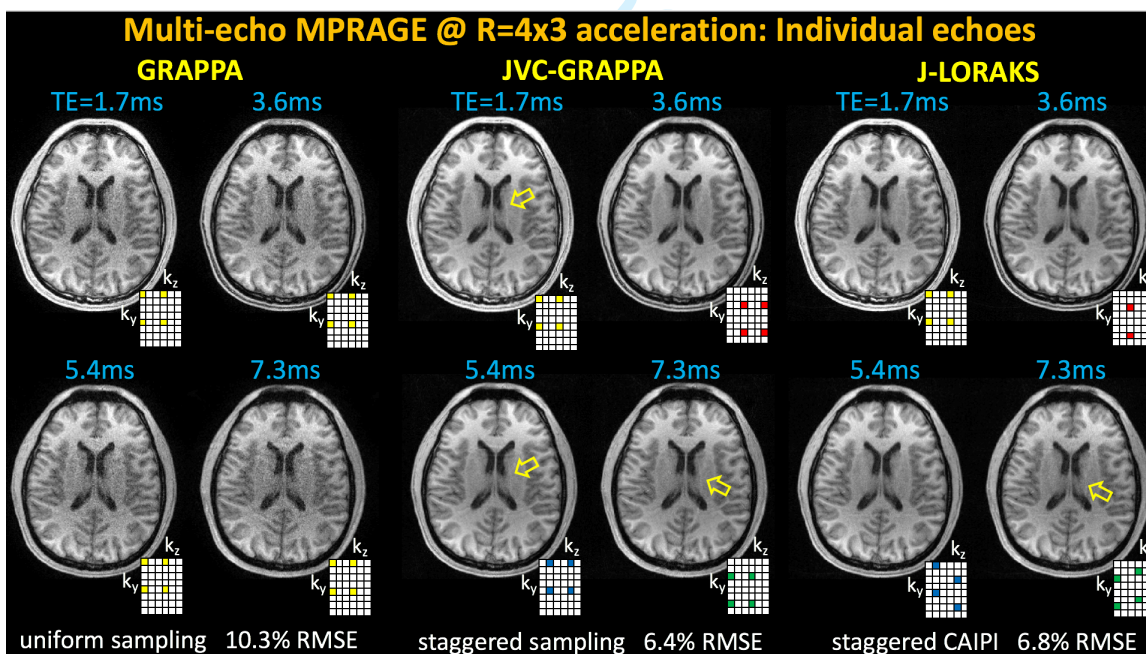
In this work, we have performed partial Fourier reconstruction for J-GRAPPA experiments using a sequential POCS processing step. It is possible to see a signal intensity underestimation issue in the estimated k-spaces of the bSSFP and MPAGE data in Figs2&4. POCS seemed more successful in completing the missing data in the TSE experiment in Fig6. We think that this is because there is less background phase in the spin-echo acquisition which rewinds most of the static phase during the contrast encoding. Because bSSFP includes contributions from free induction decay, spin- and stimulated-echoes, we expect a significant amount of high frequency spatial phase variation that is harder for POCS to estimate and incorporate into partial Fourier reconstruction. MPAGE acquisition has short TE's during which phase evolution should be small. However, within each TR, an entire plane of  $k_x$ - $k_z$  data are sampled. This causes significant  $T_1$  recovery to occur during the  $\sim 2$  sec readout duration, which leads to amplitude modulation along  $k_z$  encoding direction and complicates the relationship between k-space data and its symmetric location. We think that these relaxation and echo formation considerations caused POCS reconstruction to perform worse in bSSFP and MPAGE datasets. Partial Fourier sampling with J-LORAKS allowed us to avoid these issues since the reconstruction was performed in a single step rather than using a sequential POCS.

Another aspect of MPAGE acquisition is that, undersampling in the inner loop ( $k_z$  axis) does not the reduce scan time, since the total time is proportional to the number of outer loop ( $k_y$ ) phase encoding lines times the TR. Instead, it reduces the image blurring due to  $T_1$  relaxation, since  $k_z$  acceleration reduces the effective echo spacing in the inner loop direction. However, k-space reordering techniques can be utilized to convert  $k_z$  acceleration into actual scan time reduction (15,67) by reading out multiple  $k_y$  encodings during each TR. At this point, partial Fourier schemes could provide more flexible reordering schemes in the fast acquisition of (ME)-MPAGE data.

## Supporting Figures

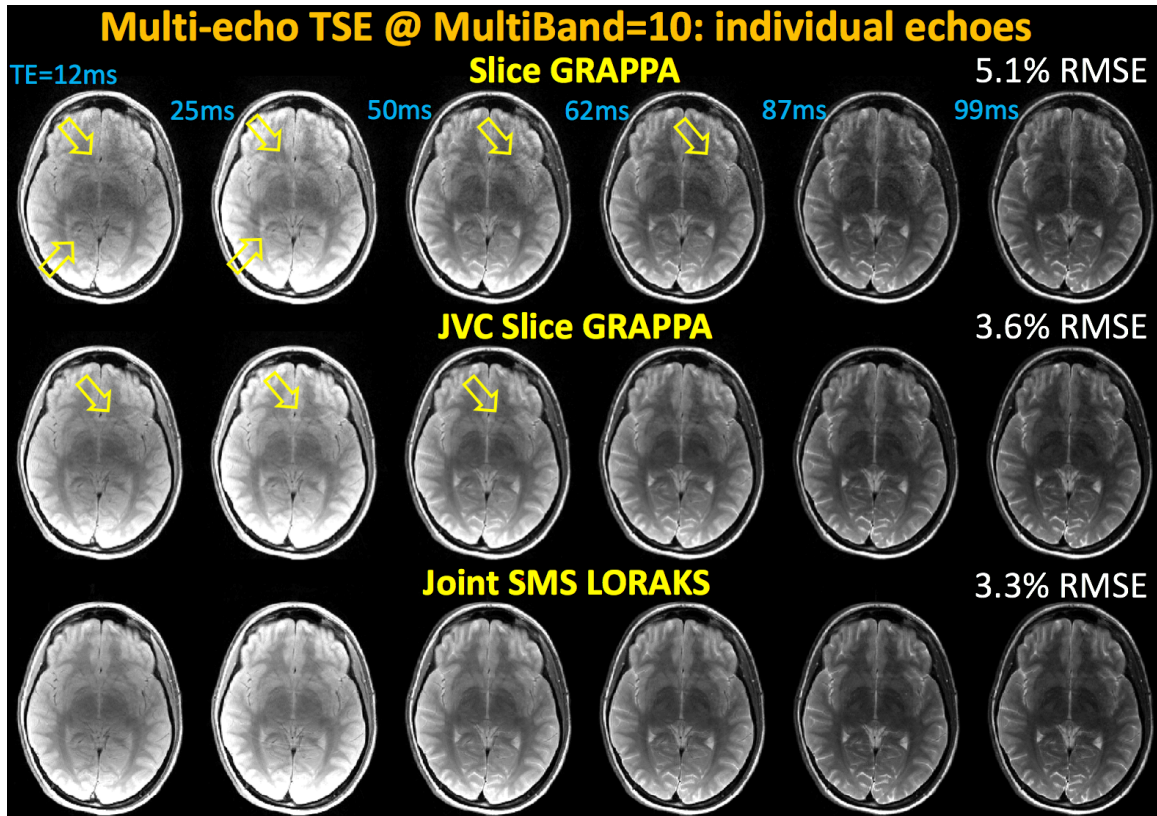


**Supporting Fig S1.** Individual bSSFP phase-cycle images at R=6-fold acceleration. K-space sampling patterns are indicated below each phase-cycle. Conventional GRAPPA incurred noise amplification and structured aliasing artifacts with 13.3% RMSE. These were largely mitigated in JVC-GRAPPA with 7.1% error, while some subtle artifacts remained (yellow arrows). J-LORAKS further improved the image quality to yield 6.5% RMSE.



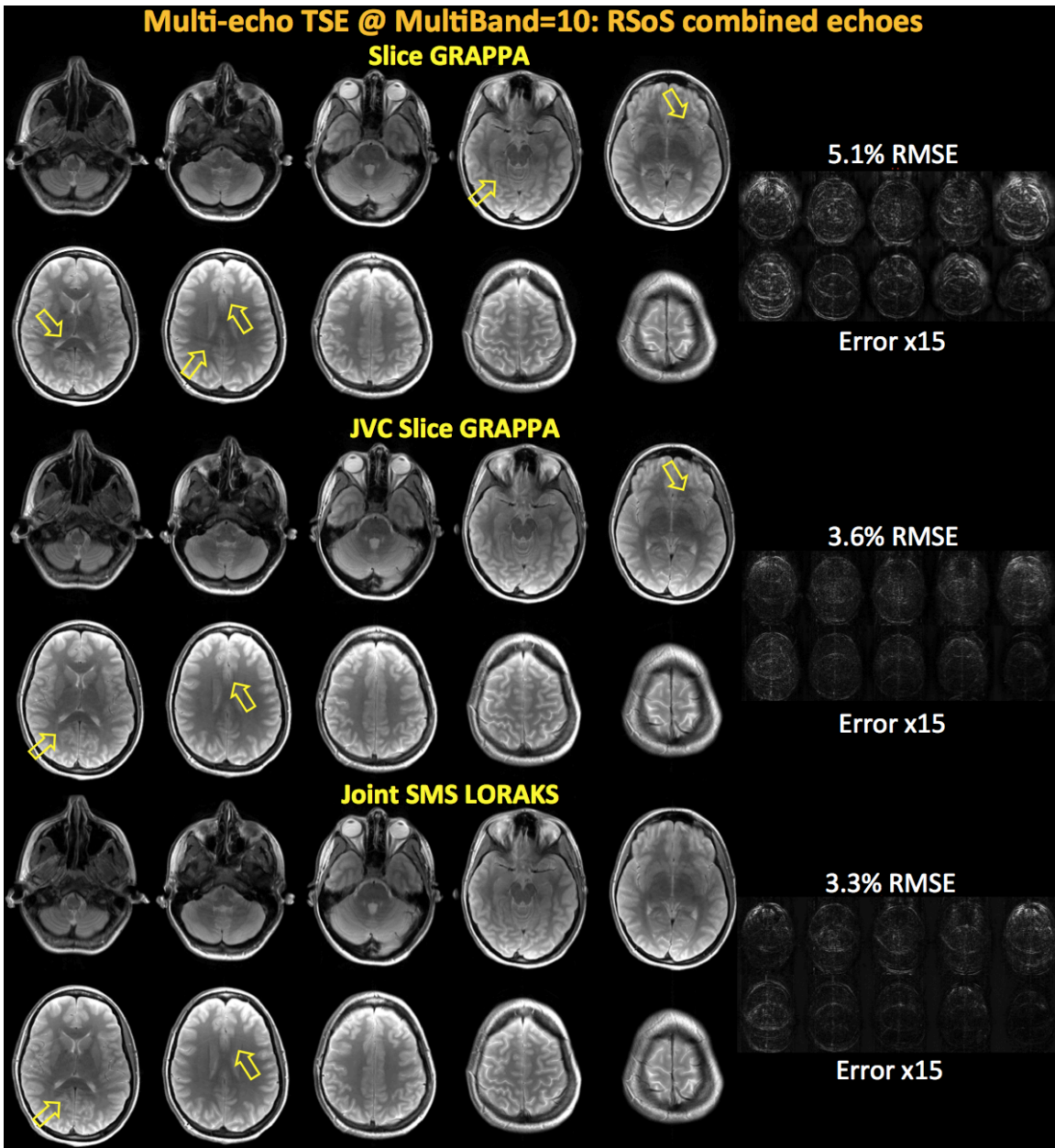
**Supporting Fig S2.** Individual echoes from the R=12-fold accelerated ME-MPRAGE reconstruction. GRAPPA with uniform sampling led to substantial noise amplification and 10.3% error. JVC-GRAPPA used staggered sampling across echoes, reduced the noise amplification and improved the RMSE to 6.4%, but suffered from some structured aliasing artifacts (yellow arrows). J-LORAKS explored the use of different CAIPI sampling patterns to mitigate both noise amplification and aliasing artifacts, with similar RMSE performance (6.8%).



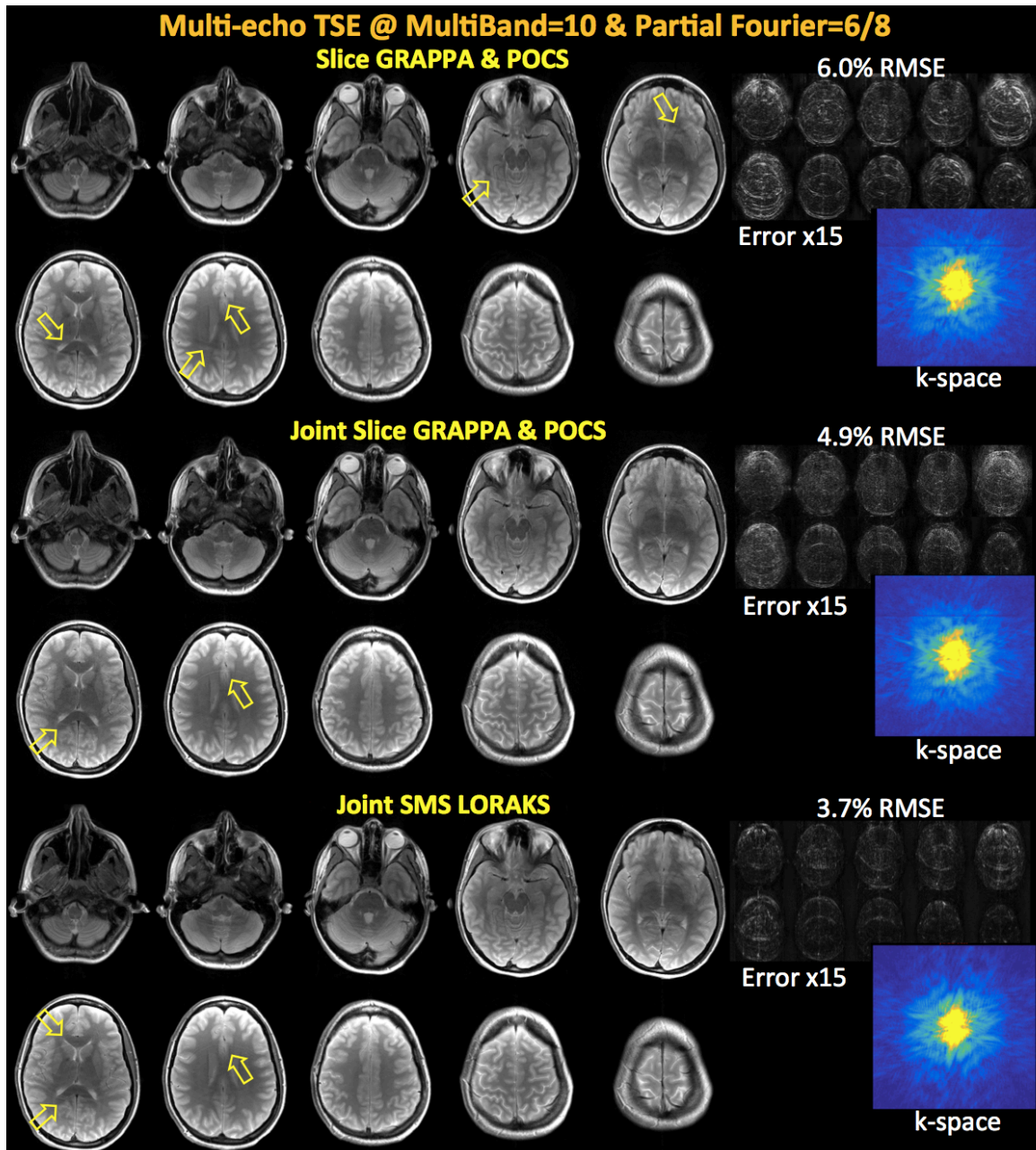


37 **Supporting Fig S3.** Individual echoes from SMS multi-echo TSE reconstruction at MB-10 acceleration. Slice  
38 GRAPPA achieved 5.1% RMSE and suffered from noise amplification and aliasing artifacts (yellow arrows), which  
39 became less apparent at the later echoes due to relaxation. JVC Slice GRAPPA was able to mitigate the noise  
40 amplification, but some of the visible aliasing artifacts remained (3.6% RMSE). Joint SMS LORAKS further improved  
41 the image quality and attained the lowest reconstruction error of 3.3%.  
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Supporting Fig S4. Multi-echo SMS TSE results at MB-10 acceleration, displaying all 10 slices from the three reconstruction techniques.



49 **Supporting Fig S5.** Multi-echo SMS TSE results at MB-10 acceleration and 6/8 Partial Fourier sampling. In addition to all 10 of the reconstructed slices, corresponding error images and k-space representation are displayed for the three algorithms.

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