Fast Reconstruction for Regularized Quantitative Susceptibility Mapping

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Declaration of Financial Interests or Relationships

Speaker Name: Berkin Bilgic

I have no financial interests or relationships to disclose with regard to the subject matter of this presentation.

Quantitative Susceptibility Mapping (QSM)

- QSM estimates the underlying magnetic susceptibility that gives rise to subtle changes in the magnetic field

- Estimation of the susceptibility map $\boldsymbol{\chi}$ from the unwrapped phase $\boldsymbol{\varphi}$ involves solving an inverse problem[1],

$$\boldsymbol{\delta} = \mathbf{F}^{-1}\,\mathbf{D}\,\mathbf{F}\,\boldsymbol{\chi}$$

measured (δ) estimate (χ)

F: Discrete Fourier Transform

D: susceptibility kernel

$\boldsymbol{\delta} = \boldsymbol{\varphi}/(\gamma \bullet TE \bullet B_0)$: normalized field map

[1] Marques JP *et al.*, Concepts in Magn Res 2005

Quantitative Susceptibility Mapping (QSM)

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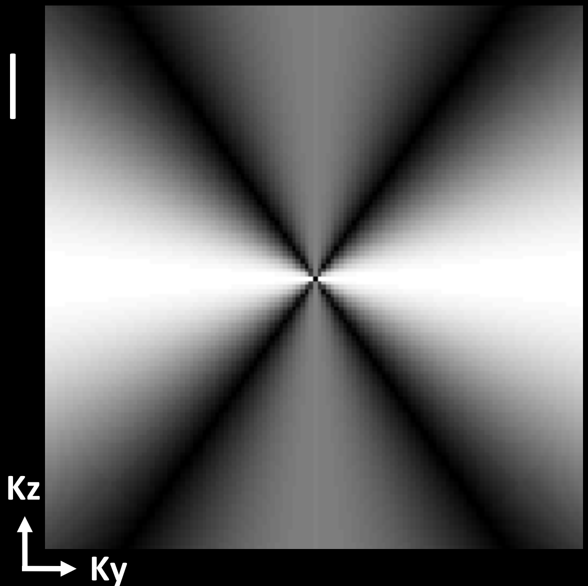
$$\boldsymbol{\delta} = \mathbf{F}^{-1}\,\mathbf{D}\,\mathbf{F}\,\boldsymbol{\chi}$$

- The inversion is made difficult by zeros in susceptibility kernel **D**

$$\mathrm{D} = \frac{1}{3} - \frac{k_z^2}{k_x^2 + k_y^2 + k_z^2}$$

|D|



Quantitative Susceptibility Mapping (QSM)

- QSM estimates the underlying magnetic susceptibility that gives rise to subtle changes in the magnetic field

- Estimation of the susceptibility map $\boldsymbol{\chi}$ from the unwrapped phase $\boldsymbol{\varphi}$ involves solving an inverse problem,

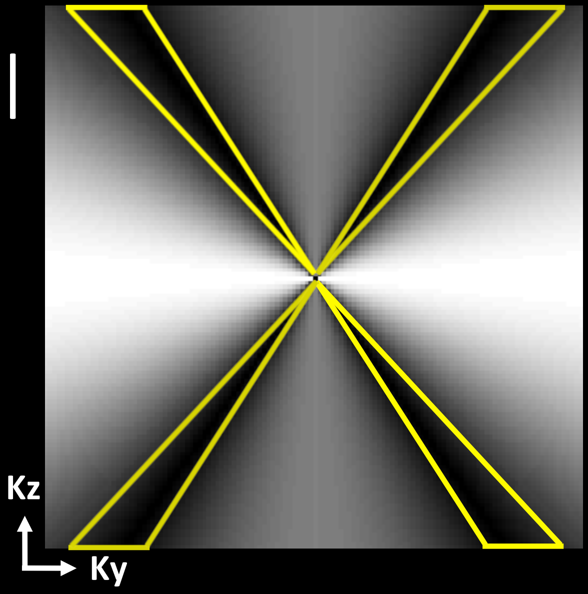
$$\boldsymbol{\delta = F^{-1} D F \chi}$$

- The inversion is made difficult by zeros in susceptibility kernel **D**

- Undersampling is due to physics

  Not in our control

|D|



Regularized Susceptibility Inversion

- Regularized QSM imposes smoothness or sparsity constraints on the gradient of the susceptibility map
- L2-regularization[1,2] (smoothness prior):

$$\min \underbrace{\left\| F^{-1} D F \chi - \delta \right\|_2^2}_{\text{Data consistency}} + \beta \cdot \underbrace{\left\| M G \chi \right\|_2^2}_{\text{Regularizer}}$$

[1] De Rochefort L *et al.*, MRM 2010 [2] Liu T *et al.*, MRM 2011

Regularized Susceptibility Inversion

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$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

G : Spatial gradient operator in 3D

M : Binary mask derived from magnitude image, prevents smoothing across edges

β : Determines the amount of smoothness

[1] De Rochefort L *et al.*, MRM 2010 [2] Liu T *et al.*, MRM 2011

Regularized Susceptibility Inversion

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- L2-regularization[1,2] (smoothness prior):

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- L1-regularization[3,4] (sparsity prior):

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| M G \chi \right\|_1$$

[1] De Rochefort L *et al*., MRM 2010 [2] Liu T *et al*., MRM 2011 [3] Liu J *et al*., NIMG 2012 [4] Wu B *et al*., MRM 2012

Regularized Susceptibility Inversion

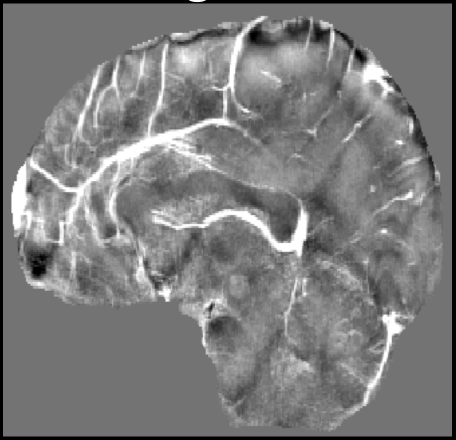
- Regularized QSM imposes smoothness or sparsity constraints on the gradient of the susceptibility map
- L2-regularization[1,2] (smoothness prior)
- L1-regularization[3,4] (sparsity prior)
- Reported reconstruction times are in the range between **20 minutes**[2,3] to **2-3 hours**[4]
- We propose efficient solvers that are up to **20× faster**
- Facilitate online recon and clinical application of QSM

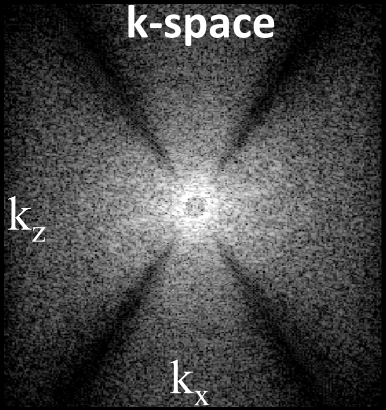
Matlab Software: martinos.org/~berkin

[1] De Rochefort L *et al*., MRM 2010 [2] Liu T *et al*., MRM 2011 [3] Liu J *et al*., NIMG 2012 [4] Wu B *et al*., MRM 2012

3D GRE 0.6 mm iso

L2 Regularized





$$\|G\chi\|_2^2$$

Closed-form L2[1]

Recon time: 0.9 sec

[1] Bilgic B *et al*., JMRI 2013

L2 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| G \chi \right\|_2^2$$

- *Without* magnitude weighting (M=Identity), we proposed a closed-form solution[1]
- This relies on computing gradients in k-space rather than image-space:

$$G = F^{-1} E F \qquad E: \text{Diagonal}$$

- With this trick, solution requires only two FFTs:

$$\chi = F^{-1} \underbrace{(D^2 + \beta \cdot E^2)}_{\text{diagonal matrix}}{}^{-1} \cdot D F \delta$$

[1] Bilgic B *et al.*, JMRI 2013

Matlab Software: martinos.org/~berkin

L2 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| G \chi \right\|_2^2$$

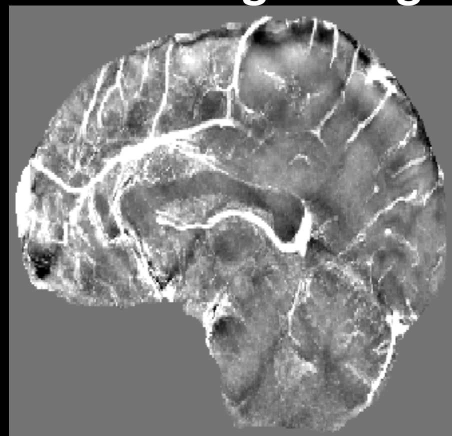
- *Without* magnitude weighting (M=Identity), we proposed a closed-form solution[1]
- This relies on computing gradients in k-space rather than image-space
- With this trick, solution requires only two FFTs
- Elegant improvements to closed-form L2: **Khabipova et al #602** and **Schweser et al #605**

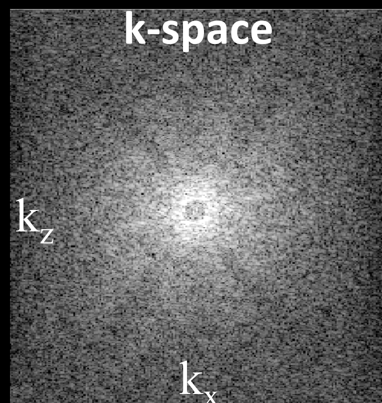
[1] Bilgic B *et al*., JMRI 2013

Matlab Software: martinos.org/~berkin

3D GRE 0.6 mm iso

L2 with Magn Weight





$$\|MG\,\chi\|_2^2$$

Proposed

Recon time: 88 sec

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$(D^2 + \beta \cdot E^H \boxed{F M F^{-1}} E) F \chi = D F \delta$$

This term cancels
if M = I

Matlab Software:
martinos.org/~berkin

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$(D^2 + \beta \cdot E^H F M F^{-1} E) F \chi = D F \delta$$

- Large linear system, solve iteratively with Conjugate Gradient (CG)

- **Proposal: Use closed-form solution to speed-up convergence of Conjugate Gradient**

Matlab Software:
martinos.org/~berkin

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$\underbrace{(D^2 + \beta \cdot E^H F M F^{-1} E)}_{\text{call A}} F \chi = D F \delta$$

Matlab Software:
martinos.org/~berkin

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$A F \chi = D F \delta$$

Matlab Software:
martinos.org/~berkin

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$A F \chi - D F \delta = 0$$

- The convergence speed of CG depends on the condition number of A
- Bring A closer to being identity using a preconditioner

Matlab Software:
martinos.org/~berkin

L2 Regularization with Magnitude Weighting

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \beta \cdot \left\| M G \chi \right\|_2^2$$

- When magnitude weighting is included, optimizer is given by the solution of:

$$\underbrace{(D^2 + \beta \cdot E^2)^{-1}}_{\text{closed-form}} \cdot (A F \chi - D F \delta) = 0$$

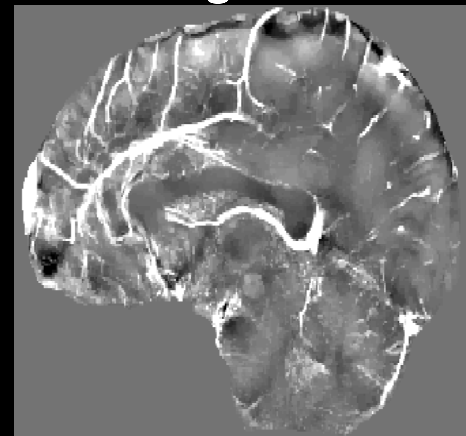
- Approximation: $(D^2 + \beta \cdot E^2)^{-1} \approx A^{-1}$

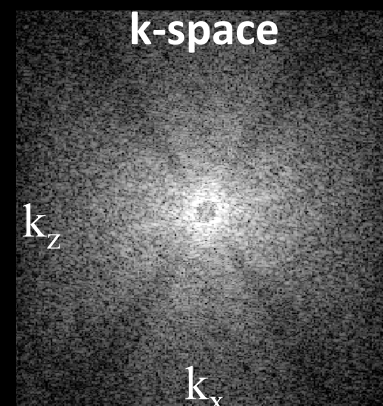
- **Preconditioned CG allows fast L2-regularization with Magnitude Weighting**

Matlab Software:
martinos.org/~berkin

3D GRE 0.6 mm iso

L1 Regularized



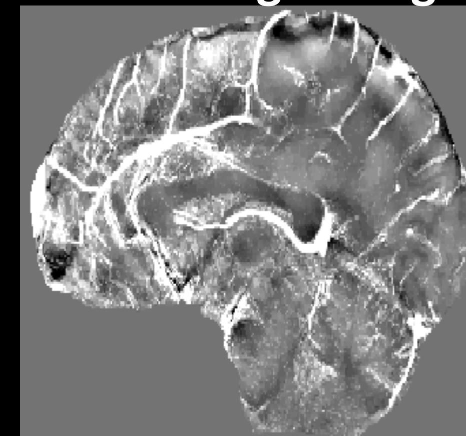


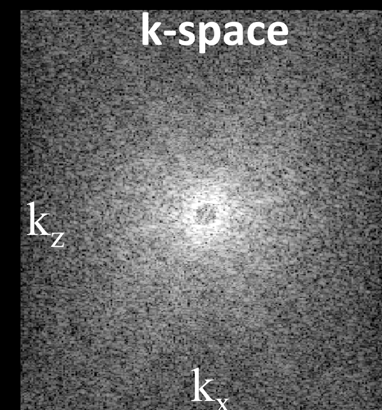
$$\|G\chi\|_1$$

Proposed

Recon time: 60 sec

L1 with Magn Weight





$$\|MG\chi\|_1$$

Proposed

Recon time: 275 sec

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| M G \chi \right\|_1$$

- L1-regularization has no closed-form solution, need to use expensive iterative methods

- **Proposal: separate L1-regularization into simpler L2-regularization and soft thresholding problems**

- **Employ closed-form solution to solve L2-problem**

Matlab Software: martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| M G \chi \right\|_1$$

Matlab Software:
martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \underbrace{\left\| y \right\|_1}_{\text{auxiliary variable}}$$

Matlab Software:
martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| y \right\|_1$$
$$\text{st} \;\; y = MG\chi$$

- Variable-splitting[1,2] separates into simpler problems

<u>1) L2-regularized:</u>

$$\chi_{t+1} = \underset{\chi}{\operatorname{argmin}} \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \mu \left\| y_t - MG\chi \right\|_2^2$$

<u>2) Soft thresholding:</u>

$$y_{t+1} = \underset{y}{\operatorname{argmin}}\, \alpha \cdot \left\| y \right\|_1 + \mu \left\| y - MG\chi_{t+1} \right\|_2^2$$

[1] Goldstein T *et al*, SIAM 2009

[2] Chen Z *et al*, J Comp Assist Tomogr 2012

Matlab Software:
martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| y \right\|_1$$

$$\text{st} \;\; y = MG\chi$$

- Variable-splitting[1,2] separates into simpler problems

1) L2-regularized:

$$\chi_{t+1} = \underset{\chi}{\operatorname{argmin}} \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \underbrace{\mu \left\| y_t - MG\chi \right\|_2^2}$$

enforces $y = MG\chi$

μ affects convergence, not final solution[1]

[1] Goldstein T *et al*, SIAM 2009 [2] Chen Z *et al*, J Comp Assist Tomogr 2012

Matlab Software: martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| \mathrm{F}^{-1}\mathrm{DF}\,\chi - \delta \right\|_2^2 + \alpha \cdot \left\| y \right\|_1$$

$$\text{st} \quad y = \mathrm{MG}\,\chi$$

- Variable-splitting[1,2] separates into simpler problems

<u>1) L2-regularized:</u>

$$\chi_{t+1} = \underset{\chi}{\operatorname{argmin}} \left\| \mathrm{F}^{-1}\mathrm{DF}\,\chi - \delta \right\|_2^2 + \mu \left\| y_t - \mathrm{MG}\,\chi \right\|_2^2$$

- Very similar to L2-regularized QSM:

$$\min \left\| \mathrm{F}^{-1}\mathrm{DF}\,\chi - \delta \right\|_2^2 + \beta \left\| \mathrm{MG}\,\chi \right\|_2^2$$

Use Preconditioned Conjugate Gradient

[1] Goldstein T *et al*, SIAM 2009

[2] Chen Z *et al*, J Comp Assist Tomogr 2012

Matlab Software: martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| y \right\|_1$$

$$\text{st} \quad y = MG\chi$$

- Variable-splitting[1,2] separates into simpler problems

2) Soft thresholding:

$$y_{t+1} = \underset{y}{\operatorname{argmin}}\, \alpha \cdot \left\| y \right\|_1 + \mu \left\| y - MG\chi_{t+1} \right\|_2^2$$

- Closed-form solution with point-wise operations:

$$y_{t+1} = \max\left(\left| MG\chi_{t+1} \right| - \frac{\alpha}{2\mu}, 0 \right) \cdot \text{sign}(MG\chi_{t+1})$$

[1] Goldstein T *et al*, SIAM 2009 [2] Chen Z *et al*, J Comp Assist Tomogr 2012

Matlab Software: martinos.org/~berkin

L1 Regularized QSM

$$\min \left\| F^{-1} D F \chi - \delta \right\|_2^2 + \alpha \cdot \left\| y \right\|_1$$

$$\text{st} \quad y = MG\chi$$

- Variable-splitting[1,2] separates into simpler problems

1) L2-regularized:

Use Preconditioned CG

Iterate until converged

2) Soft thresholding:

$$y_{t+1} = \max\left(\left| MG\chi_{t+1} \right| - \frac{\alpha}{2\mu}, 0 \right) \cdot \text{sign}(MG\chi_{t+1})$$

[1] Goldstein T *et al*, SIAM 2009

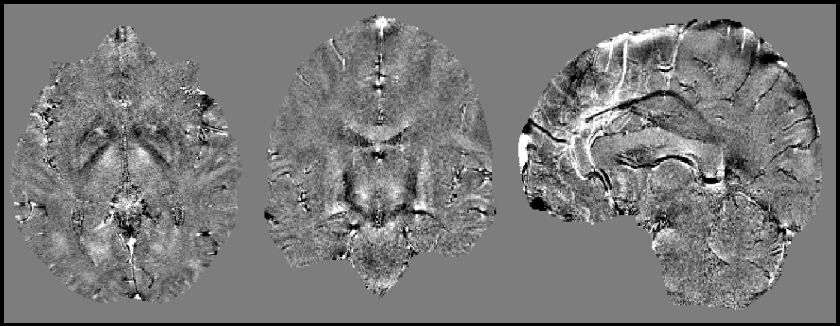
[2] Chen Z *et al*, J Comp Assist Tomogr 2012

Matlab Software: martinos.org/~berkin

Data Acquisition

❖ **High-resolution 3D GRE**

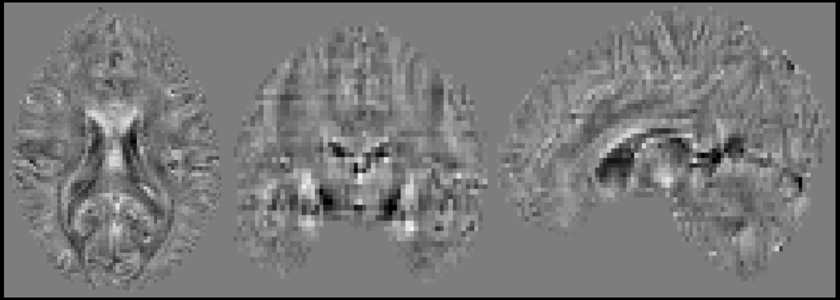
- 0.6 mm isotropic at 3T
- TR / TE = 26 / 8.1 ms
- $R_{inplane}$ = 2, Partial Fourier = 3/4
- T_{acq} = 16 min



3D GRE Phase @ 3T

❖ **Simultaneous Multi-Slice EPI**

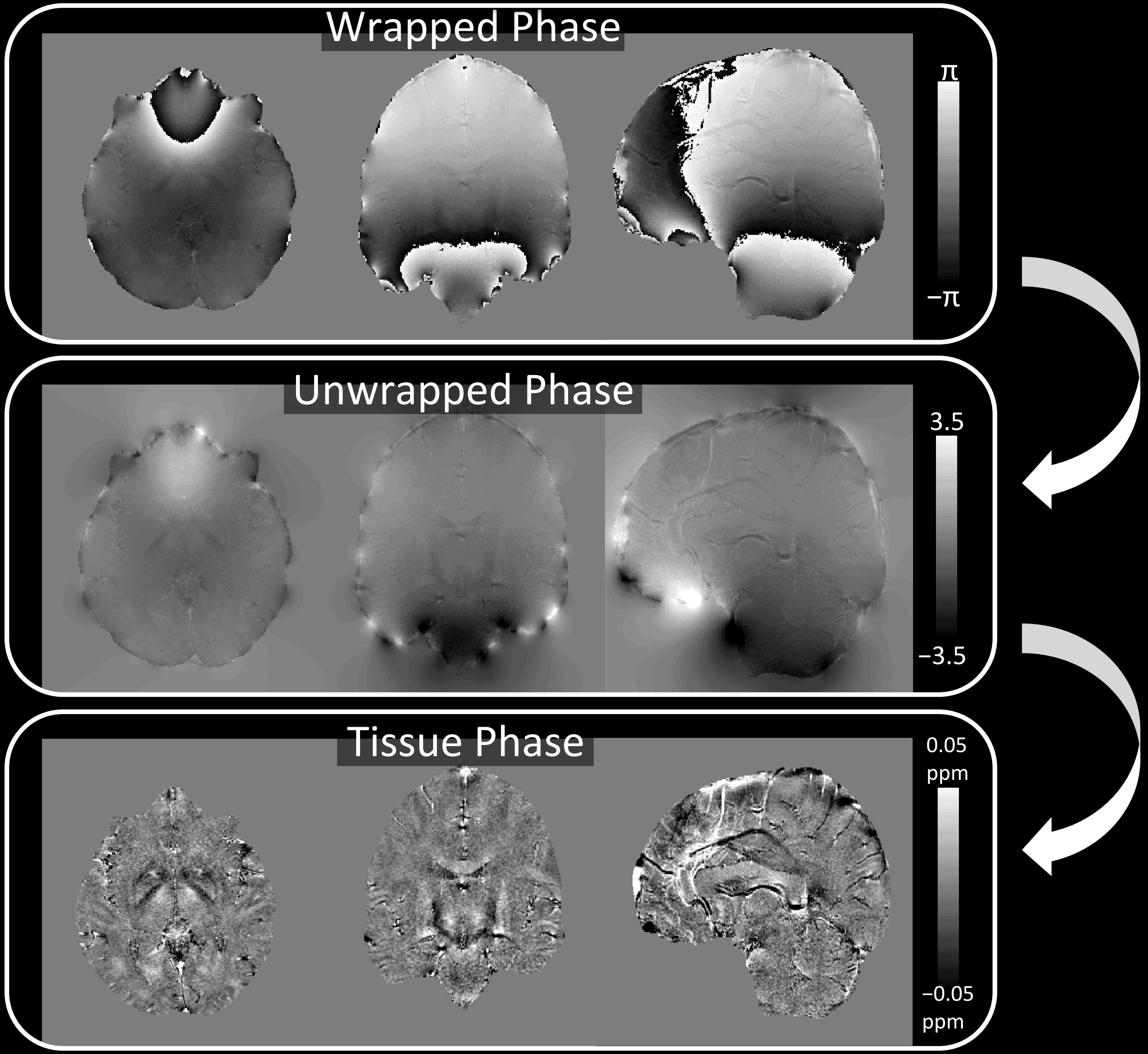
- 2 mm isotropic at 7T
- $TR/TE_1/.../TE_4$ = 2040/15/74 ms
- $R_{inplane}$ = 3, Multi-Band = 3
- T_{acq} = 2 sec



SMS EPI Phase @ 7T

Matlab Software: martinos.org/~berkin

Phase Processing   3D GRE 0.6 mm iso



Laplacian unwrapping[1]: **6 seconds**

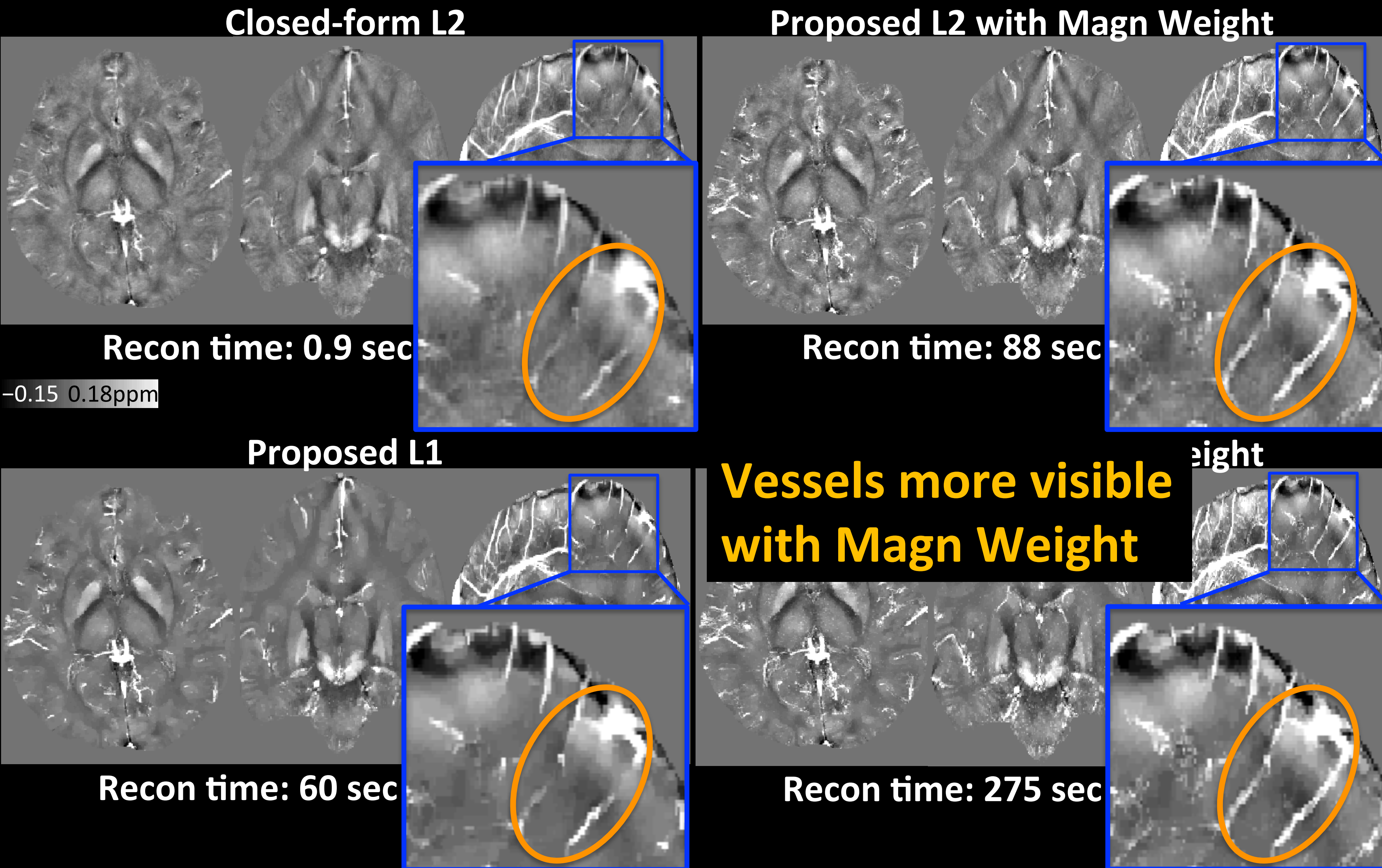
SHARP filtering[2]: **7 seconds**

Matlab Software: martinos.org/~berkin

[1] Li W *et al*, Neuroimage 2012

[2] Schweser F *et al*, Neuroimage 2011

Regularized QSM   3D GRE 0.6 mm iso



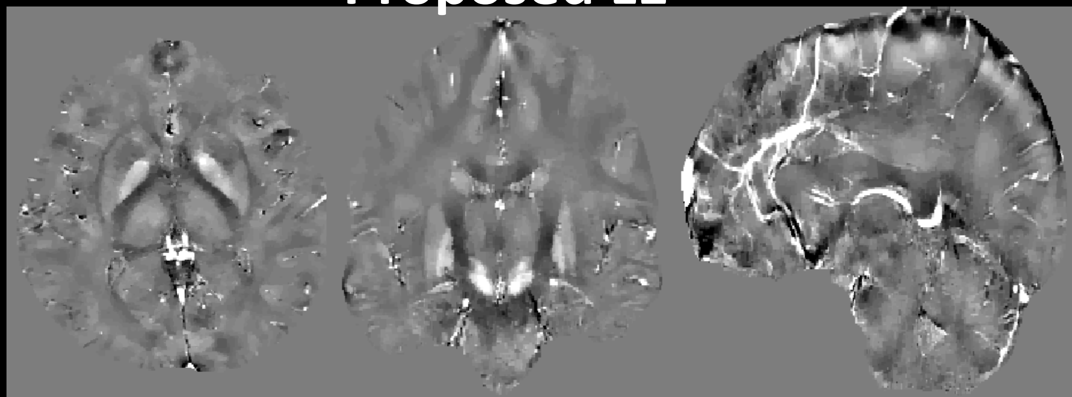
Max Intensity Projections over 3mm Slab

Matlab Software:
martinos.org/~berkin

Comparing L1-Regularized QSM Methods

3D GRE 0.6 mm iso

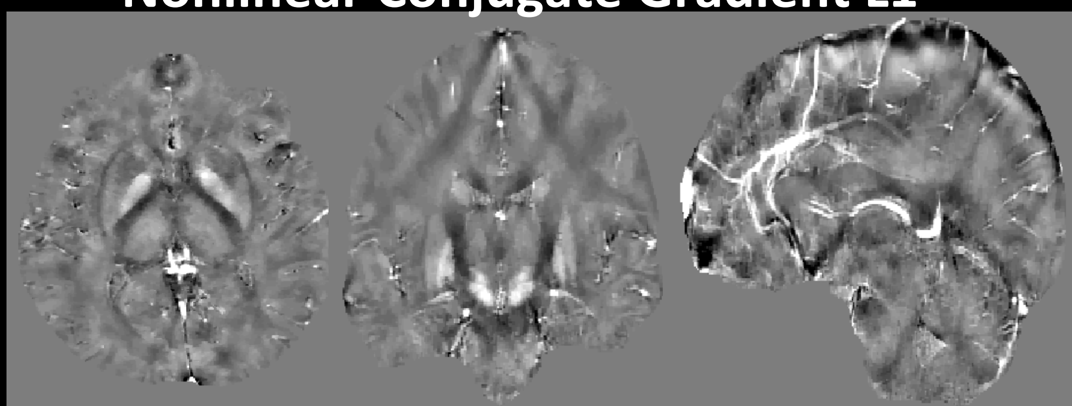
Proposed L1



Recon time : **60 sec, 13 iterations**

20× speed-up

Nonlinear Conjugate Gradient L1 [1,2]



Recon time : **1350 sec, 50 iterations**

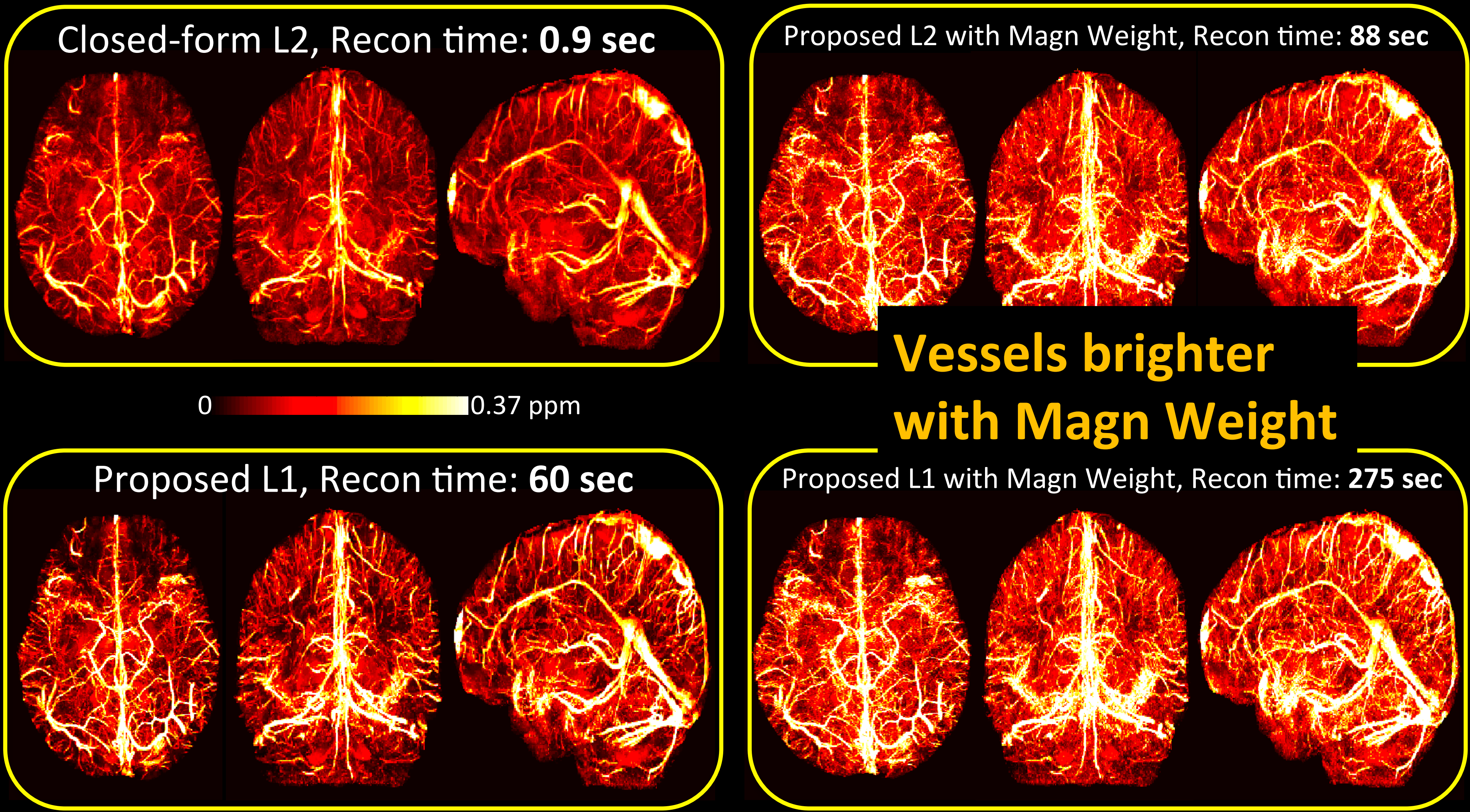
Matlab Software: martinos.org/~berkin

[1] Lustig M *et al*, MRM 2007

[2] Bilgic B *et al*, Neuroimage 2012

Maximum Intensity Projections

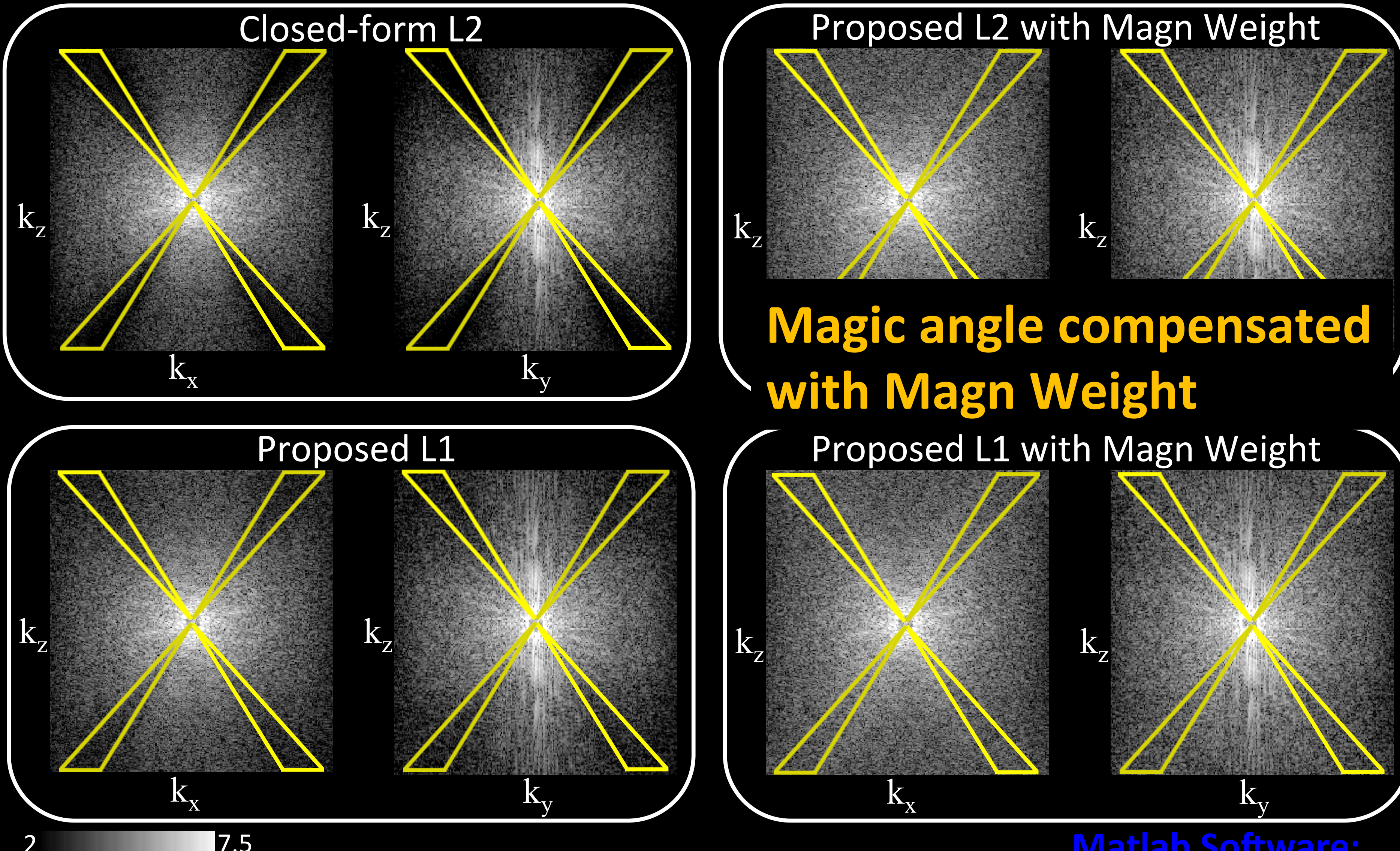
3D GRE 0.6 mm iso



Matlab Software: martinos.org/~berkin

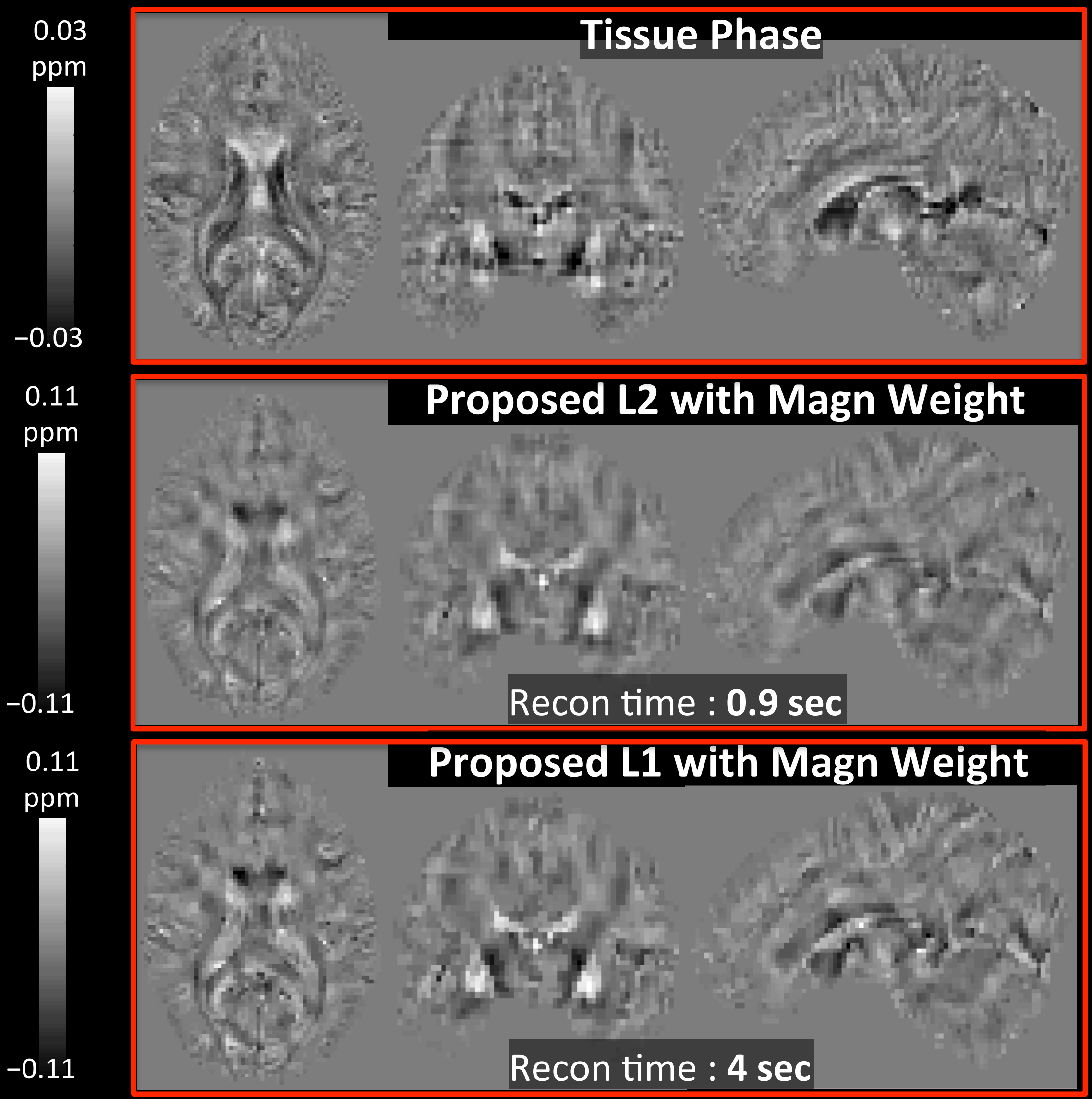
K-Space View in log scale

3D GRE 0.6 mm iso



Matlab Software: martinos.org/~berkin

SMS EPI, 2 mm iso @ 7T
$R_{inplane} = 3$, Multi-Band = 3
2 second acquisition



Fast recon may facilitate functional QSM[1,2]

[1] Balla D *et al*, ISMRM 2012

[2] Bianciardi M *et al*, HBM 2013

Fast Regularized QSM: Conclusion

- Proposed rapid L1- and L2-regularized QSM algorithms that yield up to **20× speed-up**
- Extended these to admit magnitude weighted regularization for improved reconstruction
- When combined with fast phase processing methods, these may facilitate online recon and clinical QSM

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NIH Blueprint for Neuroscience 1U01MH093765
(Human Connectome Project)

**Matlab Software:
martinos.org/~berkin**