Fast Regularized Reconstruction Tools for QSM and DSI

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Software Download: http://web.mit.edu/berkin/www/software.html

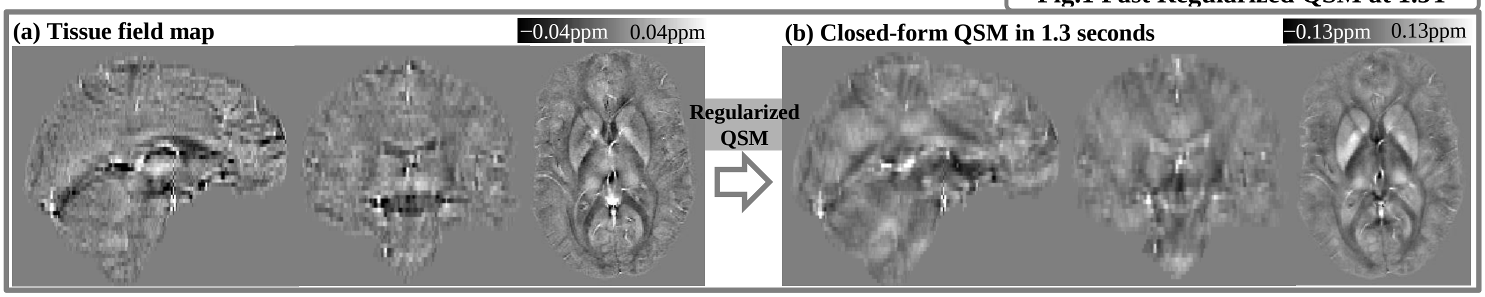
ℓ_2-regularized Reconstruction: admits closed-form solutions that can be computed efficiently. Herein, Matlab tools that achieve dramatic computational speed-up relative to iterative algorithms while retaining good reconstruction performance are presented. Two representative applications of these developed tools, Quantitative Susceptibility Mapping (QSM) and Diffusion Spectrum Imaging (DSI), are demonstrated.

I. QSM yields a map of the tissue magnetic susceptibility, $\boldsymbol{\chi}$, that lends itself to applications such as estimation of tissue iron concentration and venous oxygenation. The mapping requires the solution of an inverse problem of the form $\mathbf{F}^H\mathbf{D}\mathbf{F}\boldsymbol{\chi} = \boldsymbol{\phi}$, where $\mathbf{F}$ is the Fourier transform, $\mathbf{D}$ is a diagonal matrix with entries $1/3 - k_z^2/k^2$, $\boldsymbol{\chi}$ is the unknown susceptibility distribution and $\boldsymbol{\phi}$ is the measured tissue phase. Since the kernel $\mathbf{D}$ undersamples the frequency content of $\boldsymbol{\chi}$ along a cone in k-space, the inversion is facilitated by regularization that imposes prior knowledge [1].

Fast Regularized QSM: involves minimization of $||\mathbf{F}^H\mathbf{D}\mathbf{F}\boldsymbol{\chi} - \boldsymbol{\phi}||_2^2 + \lambda \cdot ||\mathbf{G}\boldsymbol{\chi}||_2^2$, where $\mathbf{G} = [\mathbf{G}_x; \mathbf{G}_y; \mathbf{G}_z]$ is the gradient operator in three dimensions and λ is a regularization parameter. The minimizer $(\mathbf{F}^H\mathbf{D}^2\mathbf{F} + \lambda \cdot \mathbf{G}^H\mathbf{G})^{-1}\mathbf{F}^H\mathbf{D}\mathbf{F}\boldsymbol{\phi}$ can be computed efficiently given that the matrix inversion is rapidly performed. The gradient along the x-axis can be expressed as $\mathbf{G}_x = \mathbf{F}^H\mathbf{E}_x\mathbf{F}$, where $\mathbf{E}_x$ is a diagonal matrix with entries $\mathbf{E}_x(i,i) = 1 - e^{(-2\pi\sqrt{-1}k_x(i,i)/N_x)}$, which is the k-space representation of the difference operator $\delta_x - \delta_{x-1}$. Here, k_x is the k-space index and N_x is the matrix size along x, and $\mathbf{G}_y$ and $\mathbf{G}_z$ are similarly defined. With this formulation, a closed-form solution $\widetilde{\boldsymbol{\chi}} = \mathbf{F}^H\mathbf{D}[\mathbf{D}^2 + \lambda \cdot (\mathbf{E}_x^2 + \mathbf{E}_y^2 + \mathbf{E}_z^2)]^{-1}\mathbf{F}\boldsymbol{\phi}$ is obtained. The total cost is two FFTs and multiplication of diagonal matrices.

QSM Methods and Results: 3D SPGR data were acquired on a healthy subject at 1.5T with resolution 0.94×0.94×2.5mm^3 and TR/TE = 58ms/40ms. Background phase was removed using dipole fitting [3]. λ=1.5·10^{-2} was chosen based on the L-curve heuristic. Fig.1 shows the tissue phase $\boldsymbol{\phi}$ and the $\boldsymbol{\chi}$ map reconstructed with the proposed closed-form method. For comparison, iterative conjugate gradient (CG) algorithm was employed to minimize the same objective function (results not shown). After 100 CG iterations, the root-mean-square error (RMSE) between the closed-form and iterative reconstructions was 0.3%. The processing time was 1.3 *seconds* for the closed-form solution and 29 *minutes* for the CG algorithm. Both methods are included in the QSM toolbox.

Fig.1 Fast Regularized QSM at 1.5T



II. DSI involves acquisition of the full q-space samples and yields a complete description of the diffusion probability density function (pdf) for target voxels at the expense of long imaging times (~1 hour). Significant benefit in Compressed Sensing (CS) reconstruction of DSI data from undersampled q-space was demonstrated when a dictionary trained for sparse representation was utilized [4] rather than Wavelet and Total Variation (TV) [5]. However, computation times of these CS methods are on the order of *days* for full-brain processing.

Fast Dictionary-Based DSI: In place of typical ℓ_1-regularized algorithms, two fast and simple ℓ_2-based methods are proposed. These ℓ_2-based methods rely on prior information that is built in to the dictionary and thereby forgoing the need to perform ℓ_1 optimization during the reconstruction.

i)Tikhonov regularization: Given a training set of example pdfs, K-SVD algorithm [6] is used to find a dictionary $\mathbf{D}$ that achieves sparse representation of the training pdfs. For each voxel, the following is solved: $min_{\boldsymbol{x}} \|\mathbf{F}_\Omega\mathbf{D}\boldsymbol{x} - \boldsymbol{q}\|_2^2 + \alpha \cdot \|\boldsymbol{x}\|_2^2$, where $\mathbf{F}_\Omega$ is the undersampled Fourier transform, $\boldsymbol{q}$ denote the undersampled q-space data, $\boldsymbol{x}$ are the dictionary transform coefficients, and α is a regularization parameter.

ii) Principal Component Analysis (PCA): After subtracting the average pdf $\boldsymbol{p}_{mean}$ from the training pdf dataset, PCA is applied to produce a matrix of principal pdfs $\mathbf{Q}$. A reduced-dimensionality representation is obtained by generating the matrix $\mathbf{Q}_T$ from the first T columns of $\mathbf{Q}$. The target pdf is estimated from undersampled q-space by solving: $min_{\boldsymbol{pca}} \|\mathbf{F}_\Omega\mathbf{Q}_T\boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_\Omega\boldsymbol{p}_{mean})\|_2^2$, where $\boldsymbol{pca}$ are the PCA coefficients.

DSI Methods and Results: As the minimizers in the proposed methods can be expressed in closed form, the computational cost is a single matrix-vector multiplication per voxel. Dictionary training is based on data from a subject *different* from the test subject. Optimal regularization parameters were determined using this training dataset. 515 direction, 2.3 mm isotropic DSI data with b_{max} = 8000 s/mm^2 were acquired using the 3T Connectom system. Fig.2 depicts reconstruction times and errors in the pdf space at two undersampling factors, R=3 and 9. The proposed methods demonstrate *1000-fold* speed-up relative to the previous dictionary-based algorithm in [4] with comparable reconstruction quality, while substantially reducing the reconstruction error relative to the Wavelet+TV method in [5]. Both of the proposed methods are included in the DSI toolbox.

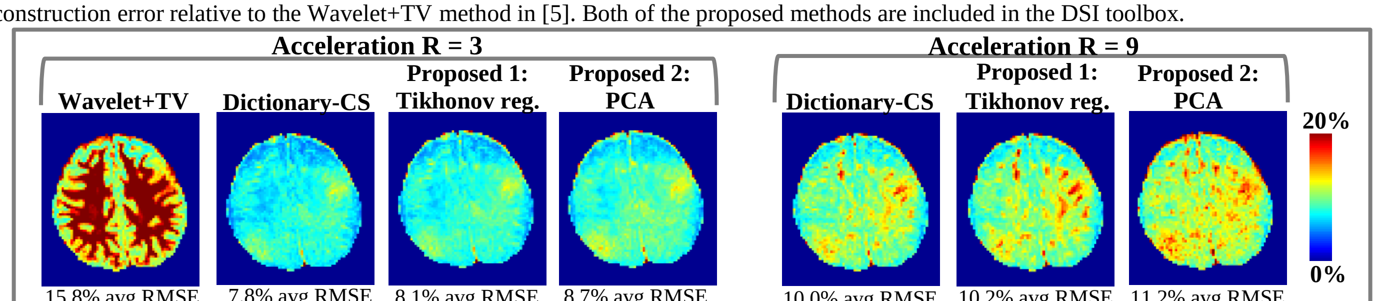


Fig.2 DSI recon: Error maps in PDF space

References: [1] de Rochefort et al. MRM'10; [2] Liu et al. NMR Biomed'11; [3] Bilgic et al. MRM'12; [4] Menzel et al. MRM'11; [5] Aharon et al. IEEE TSP'06

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