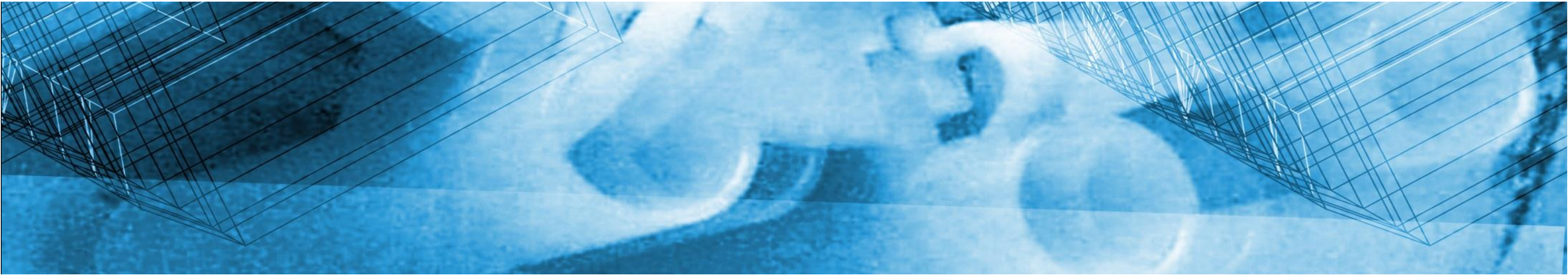

Fast Regularized Reconstruction Tools for QSM and DSI

Berkin Bilgic[1,2], Itthi Chatnuntawech[1], Kawin Setsompop[2,3], Audrey P. Fan[1], Stephen F. Cauley[2], Lawrence L. Wald[2,4], E. Adalsteinsson[1,4]

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L2-Regularized Reconstruction

- L2-regularized recon admits closed-form solutions that can be computed efficiently
- Matlab tools that achieve dramatic speed-up relative to iterative algorithms will be presented

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- L2-regularized recon admits closed-form solutions that can be computed efficiently

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- Two representative applications are considered,
    - ☐ Quantitative Susceptibility Mapping (QSM)
    - ☐ Diffusion Spectrum Imaging (DSI)

Quantitative Susceptibility Mapping (QSM)

- Quantitative Susceptibility Mapping (QSM) aims to quantify tissue magnetic susceptibility χ
- Susceptibility correlates well with tissue iron concentration, especially in iron rich deep gray matter structures [1,2]

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- Susceptibility mapping requires the solution of an inverse problem,

$$\mathbf{F}^H \underset{\substack{\swarrow \\ \text{diagonal matrix}}}{\mathbf{D}} \; \underset{\substack{\downarrow \\ \text{DFT}}}{\mathbf{F}} \; \underset{\substack{\searrow \\ \text{unknown susceptibility}}}{\chi} = \underset{\substack{\downarrow \\ \text{unwrapped phase}}}{\phi}$$

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$$\mathbf{F}^{H}\,\mathbf{D}\,\mathbf{F}\,\underset{\underset{\text{to be estimated}}{\downarrow}}{\boxed{\chi}} = \underset{\underset{\text{measured}}{\downarrow}}{\boxed{\phi}}$$

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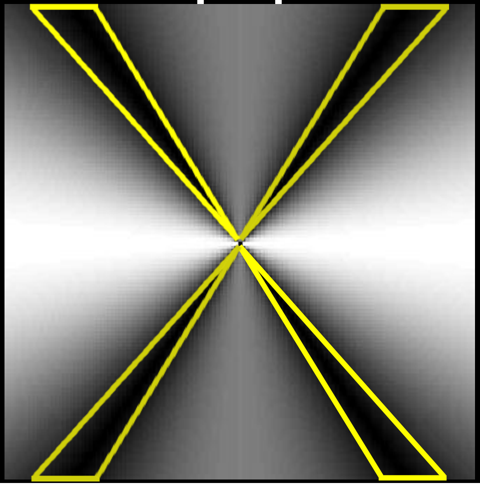
$$\mathbf{F}^H \, \mathbf{D} \, \mathbf{F} \quad \chi \;=\; \phi$$

$$\mathbf{D} = \frac{1}{3} - \frac{{k_z}^2}{k^2} \Rightarrow$$

Undersamples k-space on a conical surface

|D|



Regularized QSM

- Solution of inverse problem is facilitated by regularization that imposes prior knowledge [1]

$$\chi = argmin_{\chi} \underbrace{\|\boldsymbol{\phi} - \mathbf{F}^{H}\mathbf{D}\mathbf{F}\boldsymbol{\chi}\|_2{}^2}_{\text{data consistency}} + \lambda \cdot \underbrace{\|\mathbf{G}\boldsymbol{\chi}\|_2{}^2}_{\ell_2 \text{ over gradients}}$$

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$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{\boldsymbol{x}} \\ \mathbf{G}_{\boldsymbol{y}} \\ \mathbf{G}_{\boldsymbol{z}} \end{bmatrix}$$

gradient in 3D

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- Prior: underlying susceptibility map is smooth

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- Solution can be evaluated in closed-form

$$\chi = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

- The minimizer can be computed efficiently given that the matrix inversion is rapidly performed

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- Gradient in image space can be represented in k-space by multiplication with a diagonal matrix $\mathbf{E}$

$$\mathbf{G} = \mathbf{F}^H\mathbf{E}\ \mathbf{F}$$

where $\mathbf{E}(i,i) = 1 - e^{(-2\pi\sqrt{-1}k(i,i)/N)}$

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- $\mathbf{E}$ is simply the k-space representation of the difference operator $\delta_i - \delta_{i-1}$

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- With this formulation, closed-form solution becomes

$$\chi = \mathbf{F}^H \mathbf{D} \underbrace{[\mathbf{D}^2 + \lambda \cdot (\mathbf{E}_x^2 + \mathbf{E}_y^2 + \mathbf{E}_z^2)]^{-1}}_{\text{all matrices diagonal}} \mathbf{F} \boldsymbol{\phi}$$

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- **Total cost:** Two FFTs and multiplication of diagonal matrices

Regularized QSM Results

- Numerical Phantom
  - Three compartments (gray, white, CSF) with constant χ
  - Phase ϕ computed from true χ, and peak-SNR = 100 noise added
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- Comparison of methods
  - i. Iterative solution using Nonlinear Conjugate Gradient [1,2]
  - ii. Proposed closed-form solution

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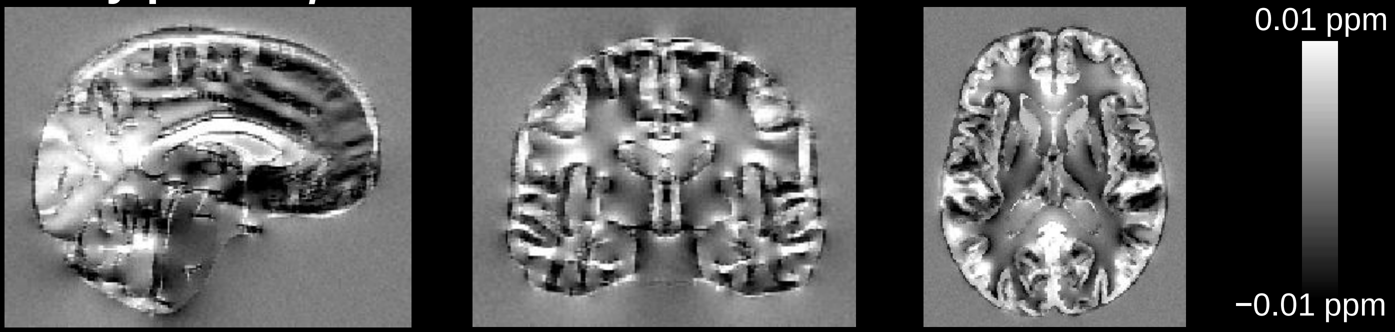
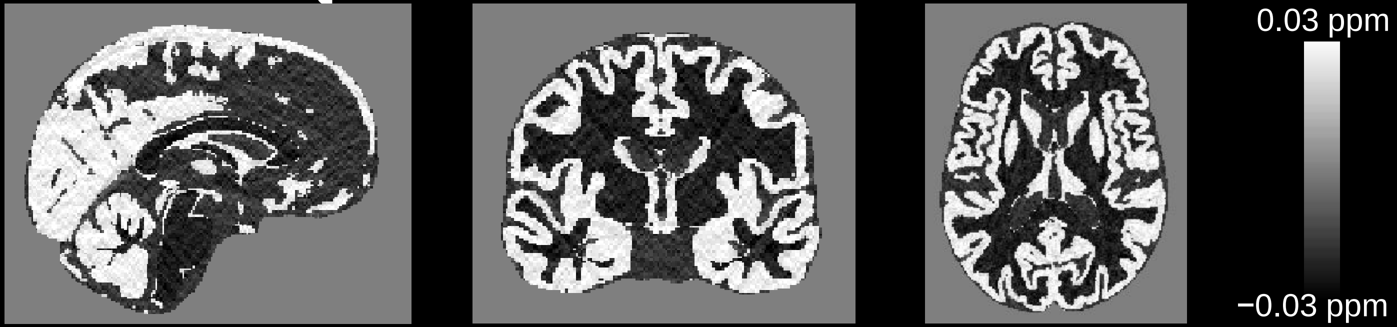
Numerical Phantom

Noisy phase ϕ

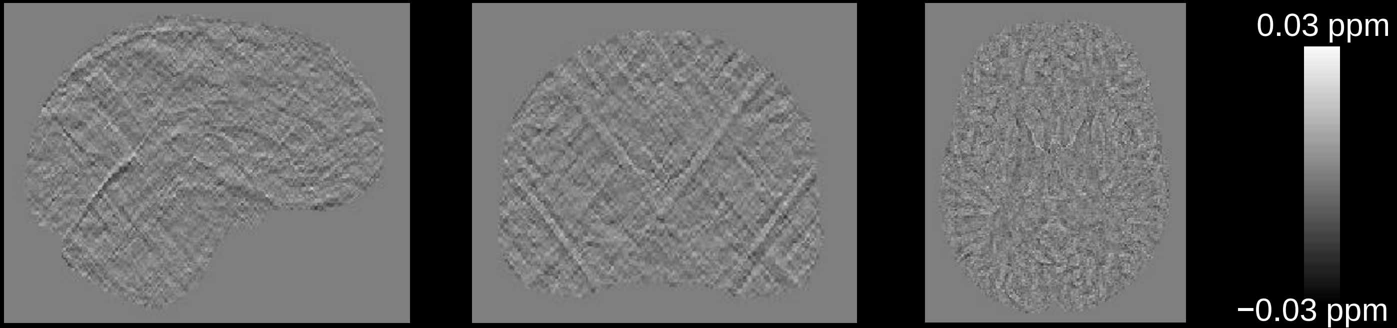
error due to noise: 5.9% RMSE



Closed-form QSM in 3.3 seconds



Closed-form QSM error relative to true χ



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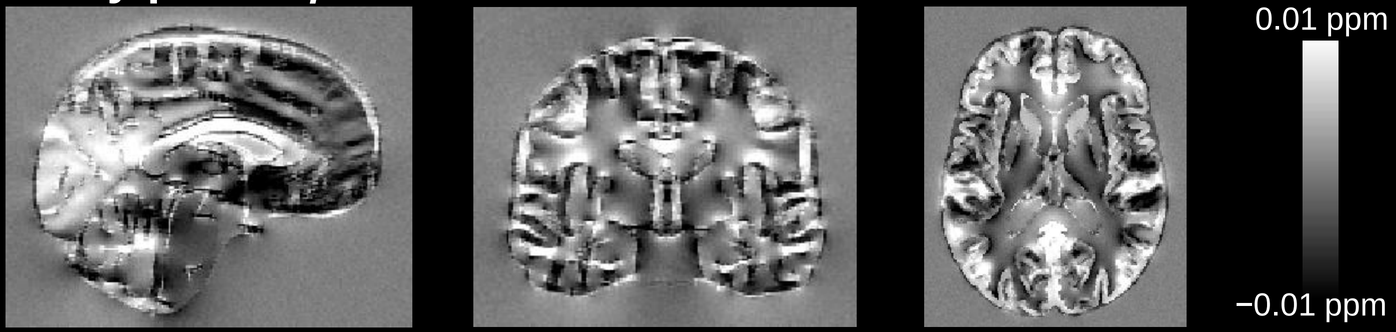

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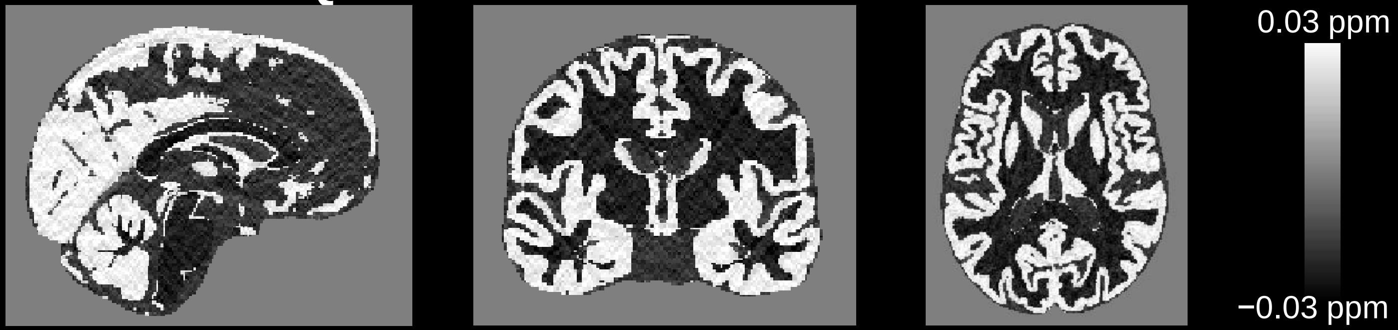
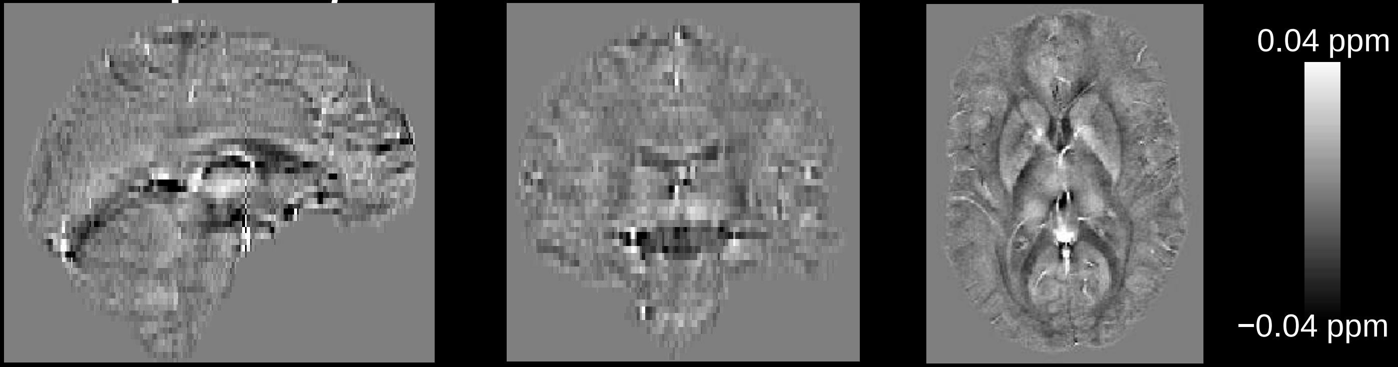


QSM Method	Recon Time	Error relative to true χ
Closed-form	**3.3 seconds**	17.4% RMSE
Iterative [1,2], 100 iters	65 minutes	18.0% RMSE

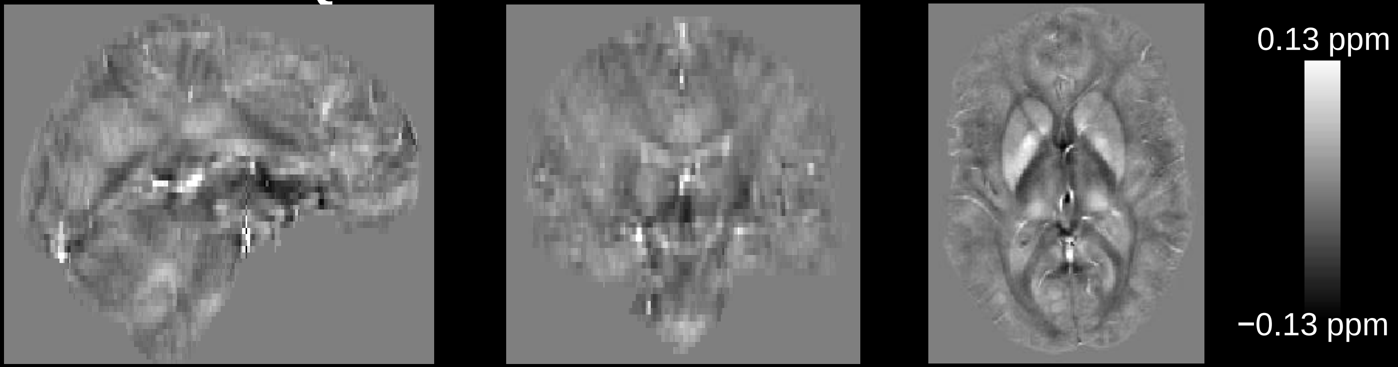
[1] de Rochefort *et al*., MRM 2010
[2] Bilgic *et al*., Neuroimage 2012

In Vivo QSM

Tissue phase ϕ



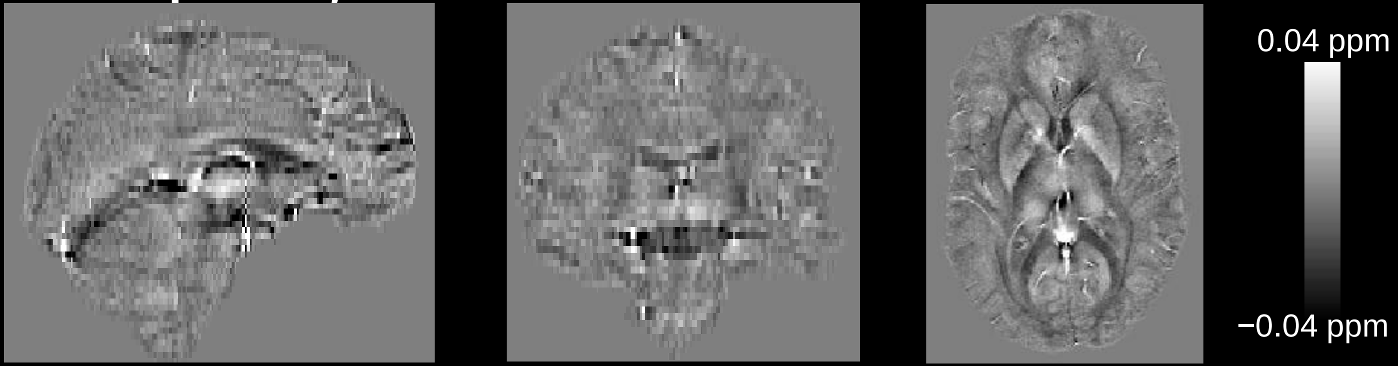
Closed-form QSM in 1.3 seconds



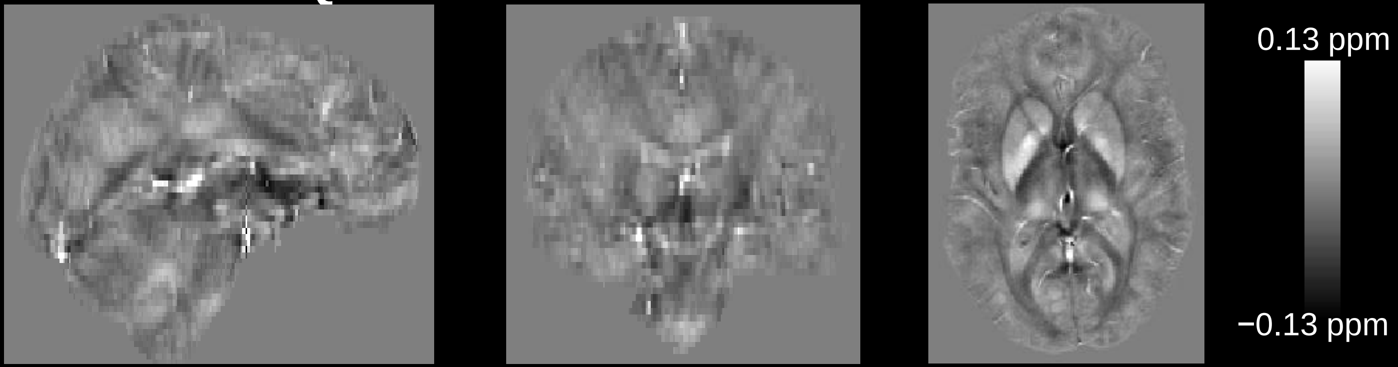
Closed-form and Iterative QSM difference

In Vivo QSM

Tissue phase ϕ



Closed-form QSM in 1.3 seconds



QSM Method	Recon Time
Closed-form	**1.3 seconds**
Iterative Conj Grad [1,2], 100 iters	29 minutes

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Diffusion Spectrum Imaging (DSI)

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- And reveals complex distributions of fiber orientations
- DSI requires full sampling of q-space (DTI needs ≥7 points)

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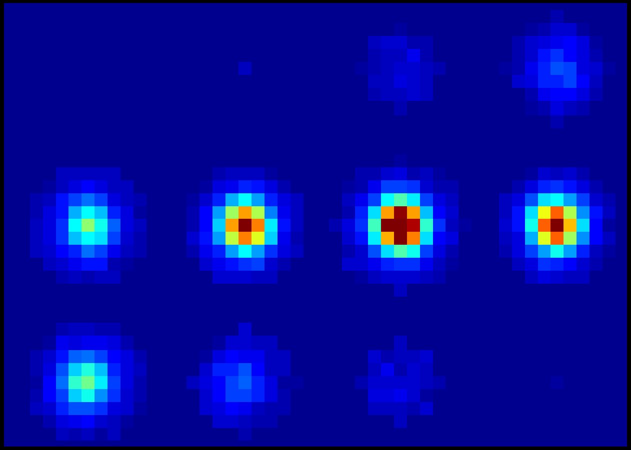
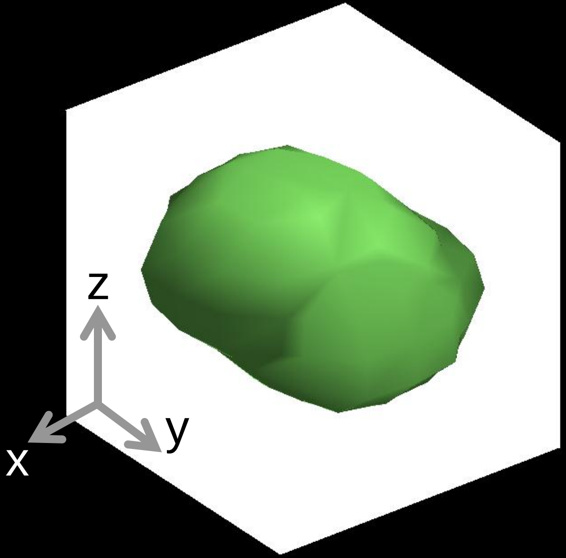
Q-space of a single voxel
515 directions



DFT

Probability Density Function (pdf)
of a single voxel

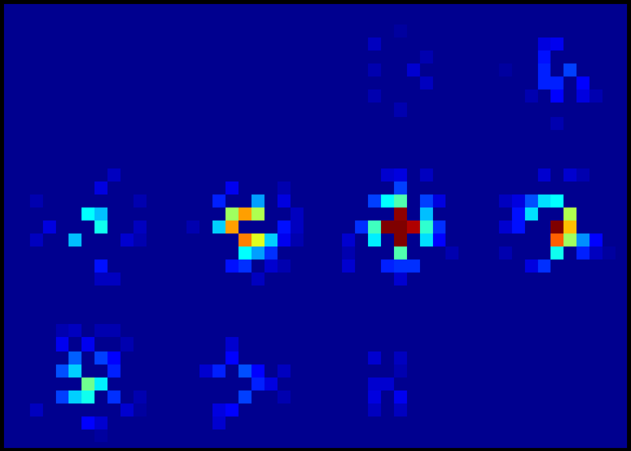


Sampling full q-space takes ~1 hour

Undersampled DSI

- To reduce scan time, undersample q-space
- Use sparsity prior to recon the pdfs via Compressed Sensing

Undersampled q-space
of a single voxel



Compressed

Sensing

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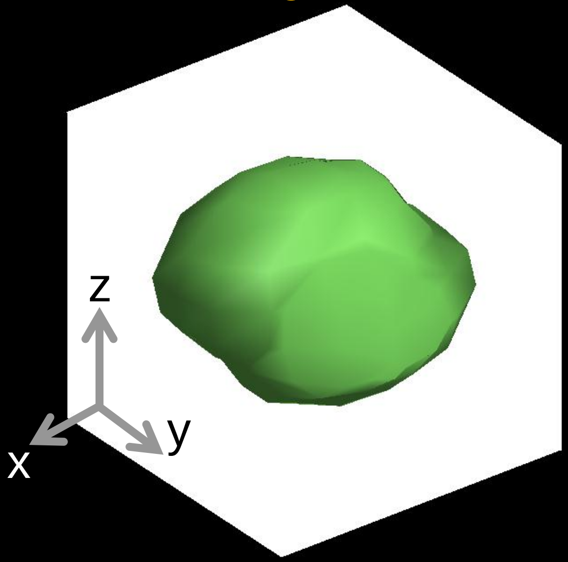

Undersampled DSI

- To reduce scan time, undersample q-space
- Use sparsity prior to recon the pdfs via Compressed Sensing
    i. Wavelet + Total Variation [1]

$$min_{\boldsymbol{p}} \, ||\mathbf{F}_{\Omega}\boldsymbol{p} - \boldsymbol{q}||_2{}^2 + \alpha \cdot ||\boldsymbol{\Psi}\boldsymbol{p}||_1 + \beta \cdot \mathrm{TV}(\boldsymbol{p})$$

undersampled DFT

pdf

q-samples

wavelet

total variation

1. Menzel *et al* MRM 2011

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ii. Dictionary-FOCUSS [2]

☐ Create a dictionary $\mathbf{D}$ from a training dataset of pdfs using K-SVD algorithm [3] → tailored for sparse representation

☐ Impose sparsity constraint via FOCUSS algorithm [4] by solving

$$min\|\boldsymbol{x}\|_1 \quad \text{such that} \quad \mathbf{F}_{\Omega}\mathbf{D}\boldsymbol{x} = \boldsymbol{q}$$

1. Menzel *et al* MRM 2011
2. Bilgic *et al* MRM 2012
3. Aharon *et al* IEEE TSP 2006
4. Gorodnitsky *et al* IEEE TSP 1997

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Dictionary transform coefficients

Iterative DSI Reconstruction

- Dictionary-FOCUSS [1] yields up to 2-times RMSE reduction using compared to Wavelet+TV

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- Full-brain recon for 10^5 voxels: ~ 10 DAYS of computation

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- Compressed Sensing recon is iterative, with processing times > 10 sec / voxel for both methods
- Full-brain recon for 10^5 voxels: ~ 10 DAYS of computation
- Two L2-based methods that are 1000-fold faster with image quality similar to Dictionary-FOCUSS are proposed:
    1. Tikhonov regularization
    2. Principal Component Analysis (PCA)

[1] Bilgic *et al* MRM 2012

Tikhonov Regularization

- Dictionary-FOCUSS iteratively solves

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- Instead, consider

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- Solution: $\widetilde{\boldsymbol{x}} = ((\mathbf{F}_\Omega \mathbf{D})^H \mathbf{F}_\Omega \mathbf{D} + \lambda \mathbf{I})^{-1} (\mathbf{F}_\Omega \mathbf{D})^H q$

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Singular Value Decomposition: $\mathbf{F}_\Omega\mathbf{D} = \mathbf{U\Sigma}\boldsymbol{V}^H$

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$$\mathbf{F}_\Omega\mathbf{D} = \mathbf{U\Sigma}V^H$$

$$\mathbf{\Sigma}^+ = (\mathbf{\Sigma}^H\mathbf{\Sigma} + \lambda\mathbf{I})^{-1}\mathbf{\Sigma}^H$$

$$\widetilde{\boldsymbol{x}} = \underbrace{\mathbf{V\Sigma}^+\mathbf{U}^H}_{\text{compute once}} q$$

PCA Reconstruction

- PCA: approximates data points using a linear combo of them to retain the maximum variance in the dataset

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- Start with a training set of pdfs $\mathbf{P}$
- Subtract the mean, diagonalize the covariance matrix:

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$$\mathbf{Z}\mathbf{Z}^H = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H$$

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- Pick the first T columns of $\mathbf{Q}$ corresponding to largest eigvals: $\mathbf{Q}_T$

$$\underset{\displaystyle\downarrow}{\boldsymbol{pca}} = \mathbf{Q}_T^{H}(\boldsymbol{p} - \boldsymbol{p}_{mean})$$

T - dimensional
pca coefficients

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- The location of $\boldsymbol{pca}$ in the pdf space,

$$\boldsymbol{p}_{T} = \mathbf{Q}_{T}\boldsymbol{pca} + \boldsymbol{p}_{mean}$$

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- Least-squares approximation in T - dimensions,

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$$min_{\boldsymbol{pca}} \ \|\mathbf{F}_{\Omega}\mathbf{Q}_{T}\boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_{\Omega}\boldsymbol{p}_{mean})\|_2^2$$

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$$min \; \|\mathbf{F}_\Omega \boldsymbol{p}_T - \boldsymbol{q}\|_2{}^2$$

- In PCA coordinates,

$$min_{\boldsymbol{pca}} \; \|\mathbf{F}_\Omega \mathbf{Q}_T \boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_\Omega \boldsymbol{p}_{mean})\|_2^2$$

- Closed-form solution:

$$\widetilde{\boldsymbol{pca}} = \underbrace{\mathrm{pinv}(\mathbf{F}_\Omega \mathbf{Q}_T)}_{\text{compute once}}(\boldsymbol{q} - \mathbf{F}_\Omega \boldsymbol{p}_{mean})$$

DSI Acquisition

- 2.3 mm isotropic with b_{max} = 8000 s/mm^2 at 3T
- Connectom gradients and 64-chan head coil [1]
- 515 q-space points collected in 50 min

1. Keil *et al* MRM 2012

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  - ii. Dictionary-FOCUSS [3]
  - iii. Tikhonov regularization
  - iv. PCA

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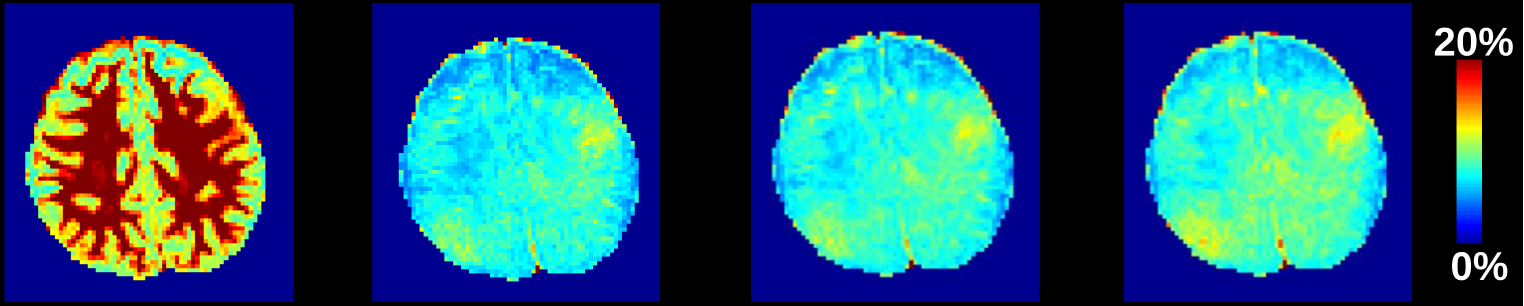
2. Menzel *et al* MRM 2011
3. Bilgic *et al* MRM 2012

pdf reconstruction error maps

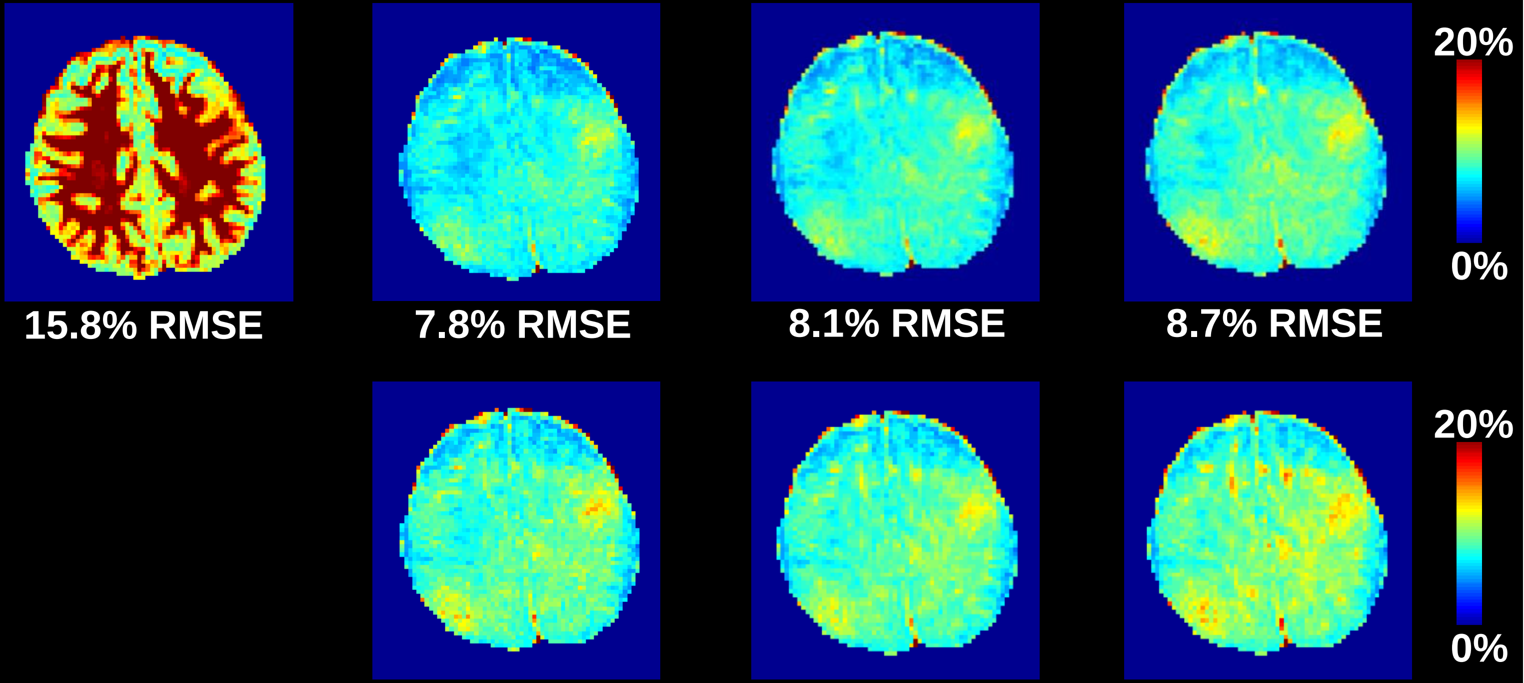
Recon Time	**1190 min**	**530 min**	**0.6 min**	**0.4 min**
	Wavelet+TV	**Dict-FOCUSS**	**Tikhonov**	**PCA**

Acceleration
R = 3



15.8% RMSE	**7.8% RMSE**	**8.1% RMSE**	**8.7% RMSE**

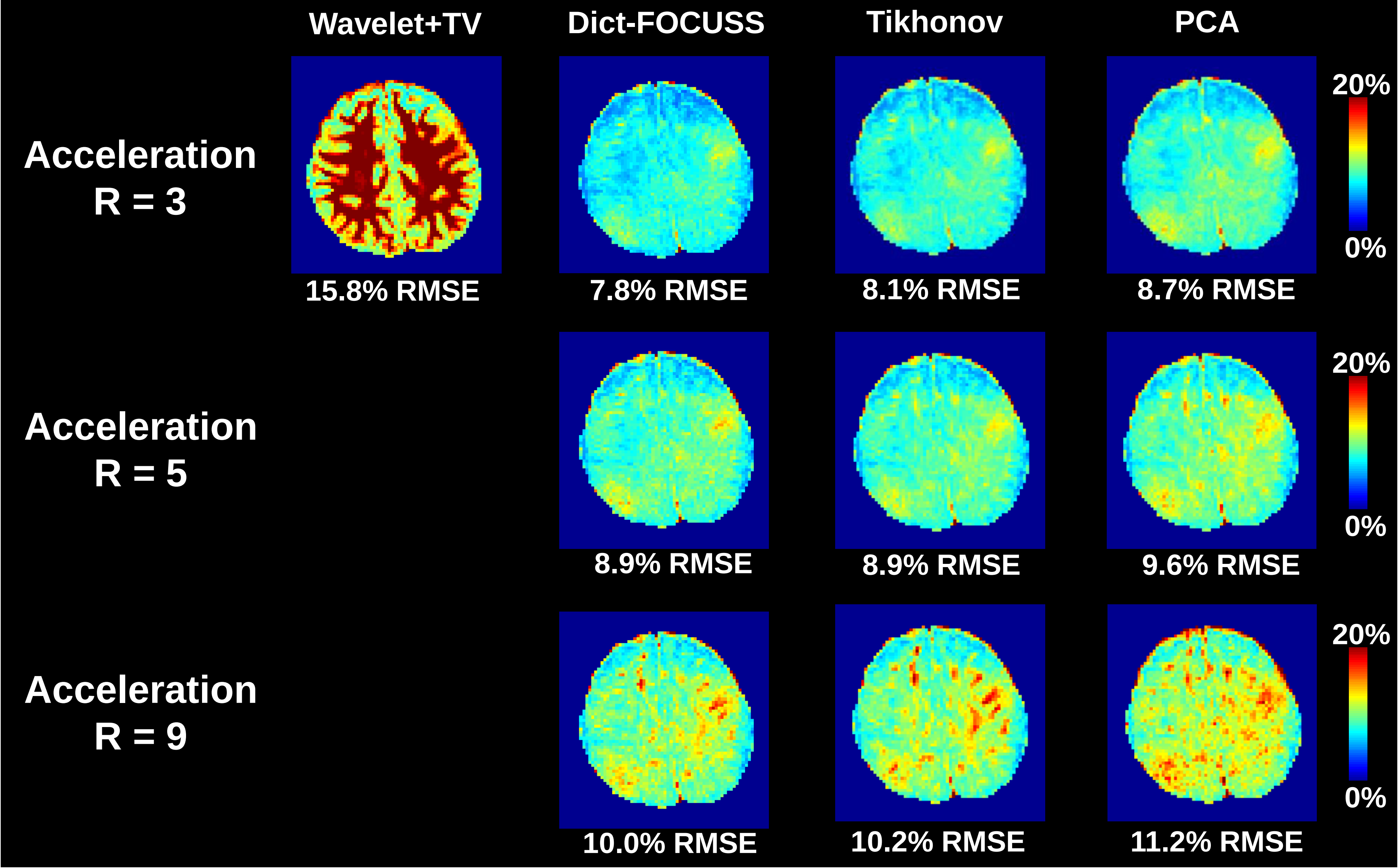
pdf reconstruction error maps



Recon Time	**1190 min**	**530 min**	**0.6 min**	**0.4 min**
	Wavelet+TV	**Dict-FOCUSS**	**Tikhonov**	**PCA**
Acceleration R = 3	15.8% RMSE	7.8% RMSE	8.1% RMSE	8.7% RMSE
Acceleration R = 5		8.9% RMSE	8.9% RMSE	9.6% RMSE

pdf reconstruction error maps

Recon Time	1190 min	530 min	0.6 min	0.4 min



Conclusion

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i. Quantitative Susceptibility Mapping

- ❑ Closed-form solution:1000-fold speed up obtained relative to state of the art

ii. Diffusion Spectrum Imaging

- ❑ Rather than enforcing sparsity, it seems that using a dictionary is the key to good reconstruction

- ❑ 1000-fold speed up obtained relative to Compressed Sensing

Software Download:

http://web.mit.edu/berkin/www/software.html